Conditional Mean Spectrum: Assessment and Refinements

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Why Conditional Mean Spectrum?

- Uniform Hazard Spectrum (UHS) has been widely used as the design (or target) spectrum in practice.

- UHS envelopes the spectral amplitudes at all vibration periods with a specified excedance probability.

- Recently proposed CMS (Baker and Cornell, 2006) is a more realistic alternative to UHS.

Objectives

- To define the “true” (or actual) CMS with given hazard conditions and SA(T_1)
- assess the current method to determine CMS
- develop an alternative procedure for CMS

- Procedure presented:
  - Based on Least Square (LSQ) Fit
  - Considers magnitude (M) and distance (R) bin if there is enough data

“Actual” CMS and How to Construct It?

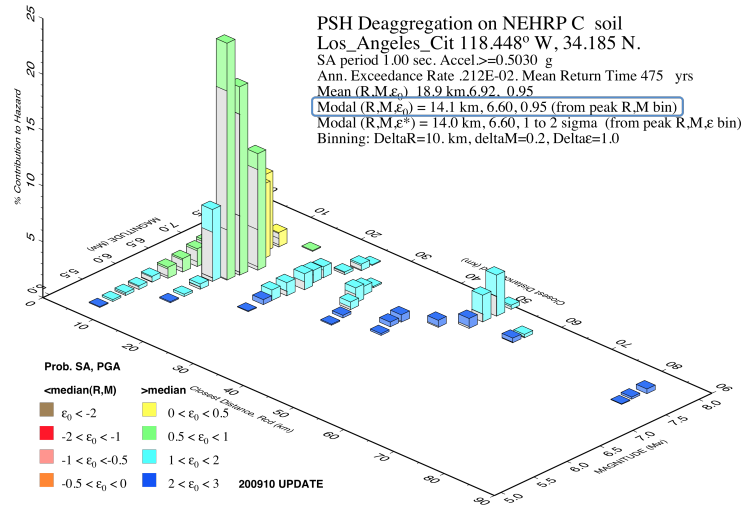
Assume existence of enough records with desired value of SA(T_1) for specified M, R and site condition:

CMS_{actual} = median spectrum of such records

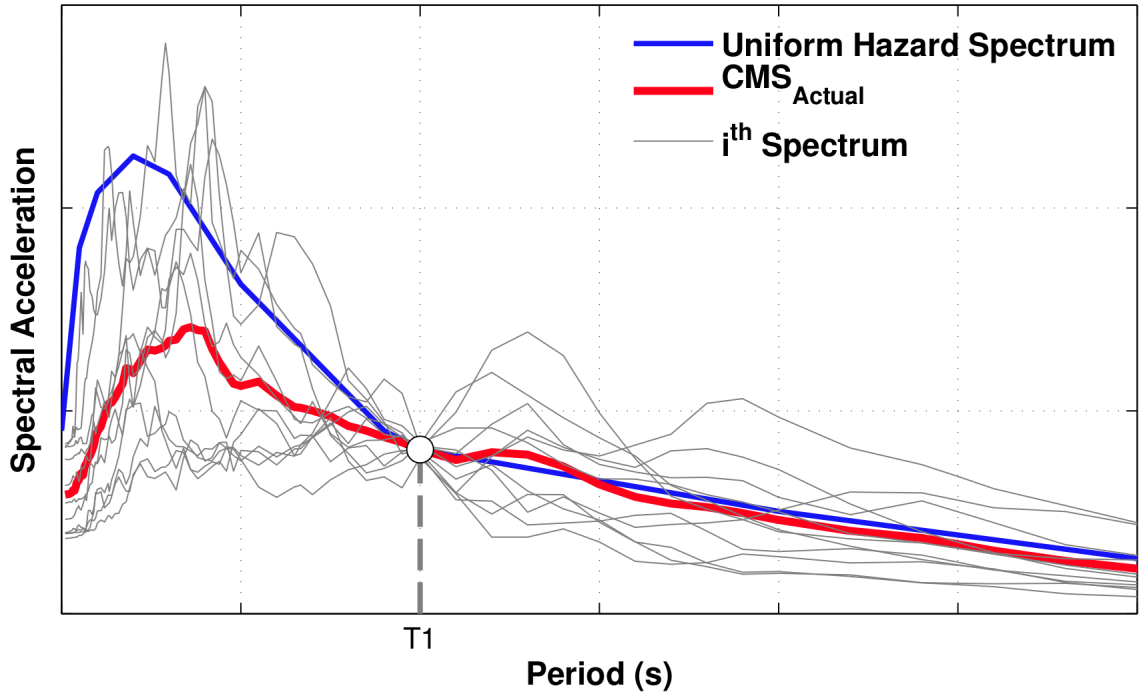
Note: GMPE is not needed

Epsilon (ε) is not used

Computation of "Actual" CMS



- ✓Magnitude
- ✓Distance
- ✓$SA(T_1)$

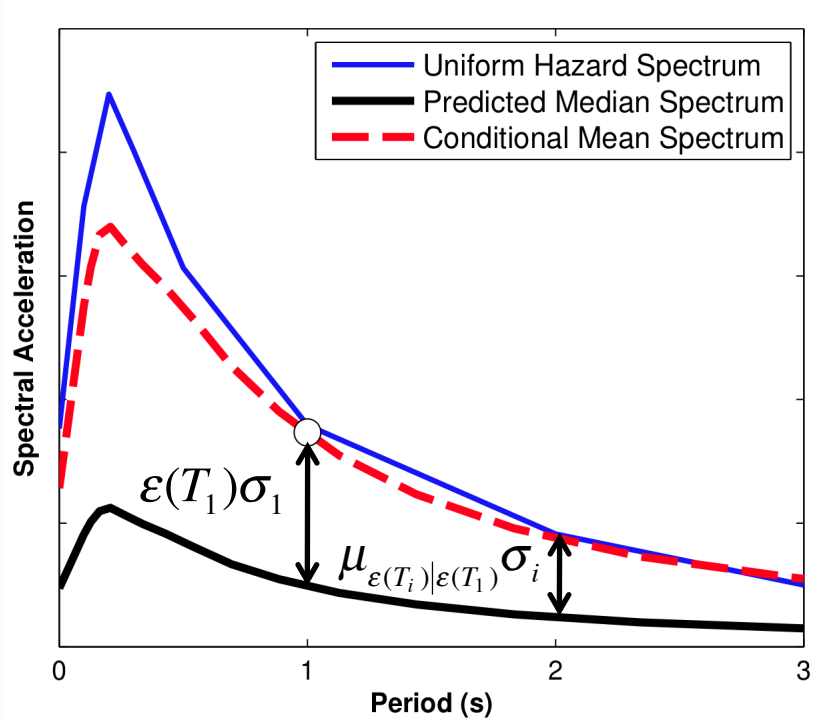


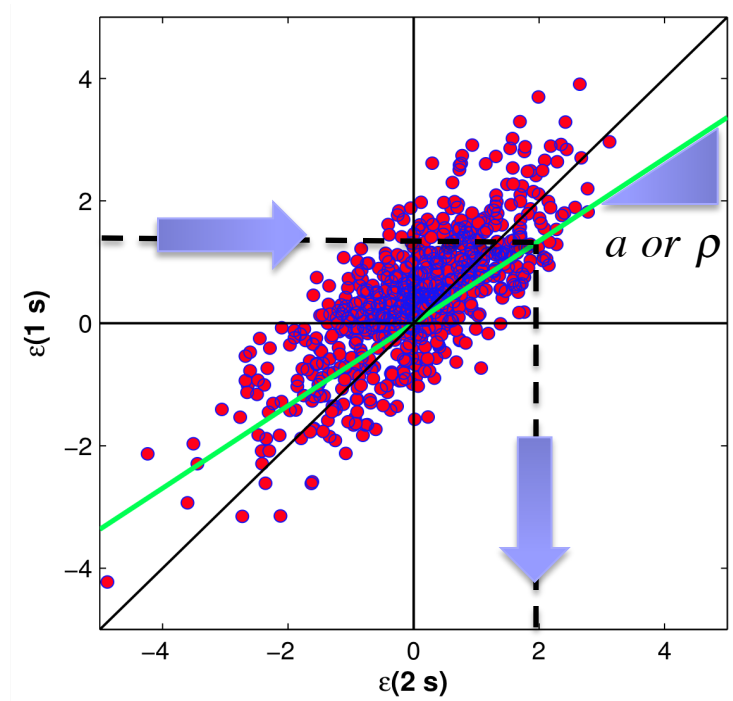
“Approximate” CMS

- Scarcity of records for large M and small R prevents construction of actual CMS

- Approximate CMS can be constructed using a GMPE and by relating $\varepsilon(T_i)$ and $\varepsilon(T_1)$
  - Correlation analyses (Baker and Cornell, 2006)
  - Regression analyses (LSQ Fit)

Regression v's Correlation Analyses





Correlation Coefficient

$$\rho_{\varepsilon(T_1),\varepsilon(T_i)} = \frac{\mathrm{cov}[\varepsilon(T_1),\varepsilon(T_i)]}{\sigma_{\varepsilon(T_1)}\sigma_{\varepsilon(T_i)}}$$

LSQ Fit

$$a_{\varepsilon(T_i)|\varepsilon(T_1)}$$

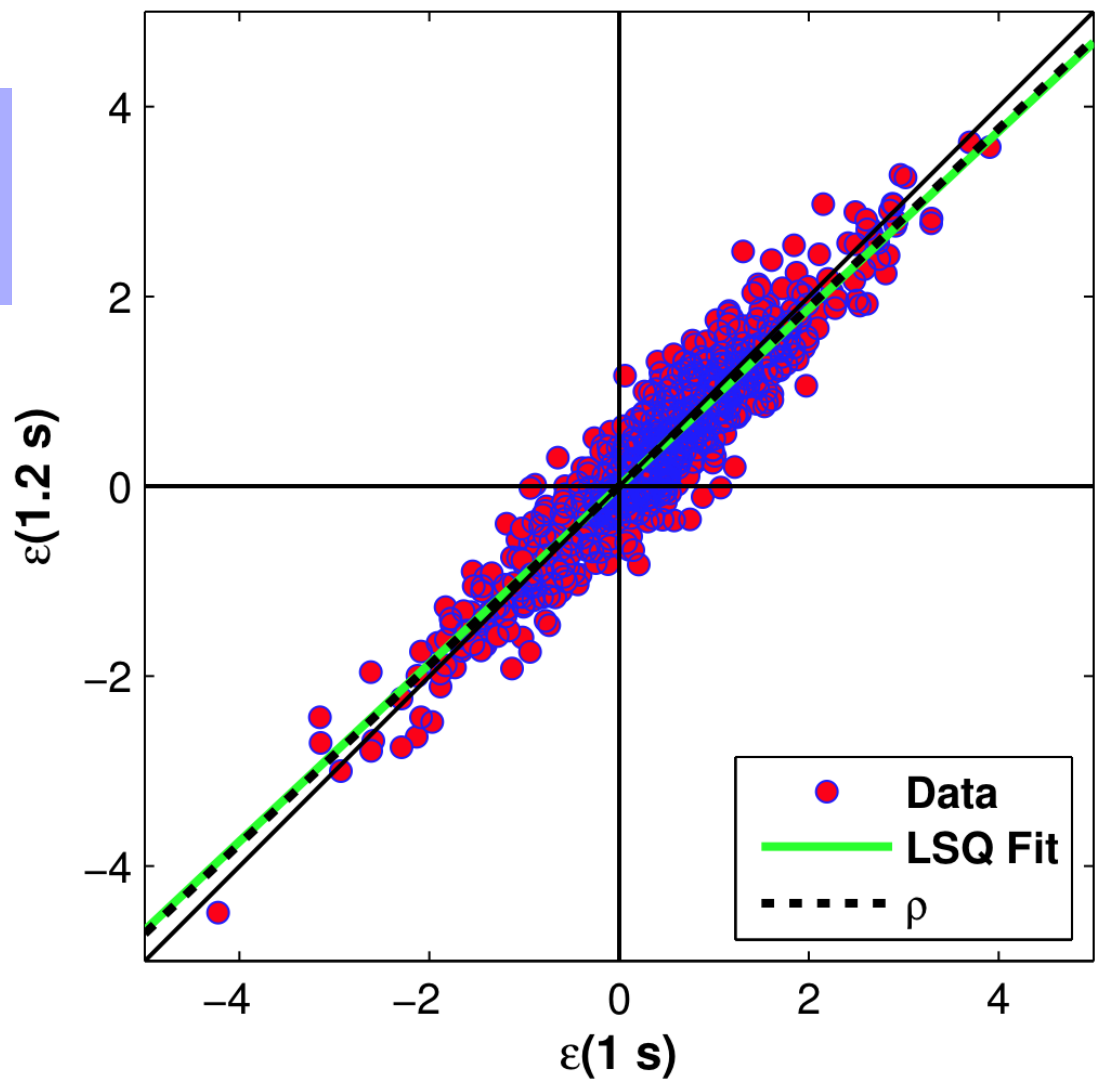
$$\mu_{\varepsilon(T_i)|\varepsilon(T_1)} = a(or\ \rho)\ \varepsilon(T_1)$$

Correlation coefficient v's LSQ Fit (1 s v's 1.2 s)

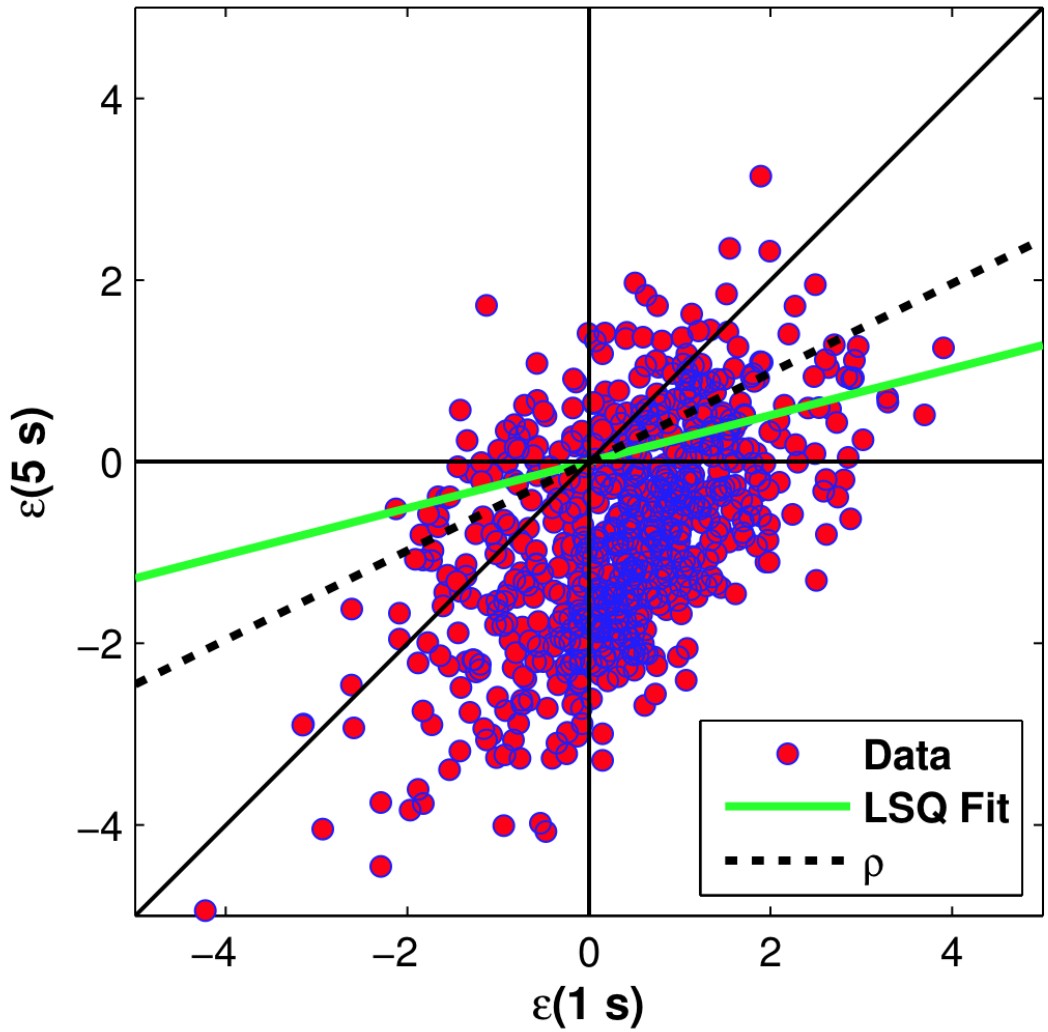
$$a_{\varepsilon(T_i),\varepsilon(T_1)} = \rho_{\varepsilon(T_1),\varepsilon(T_i)} \frac{\sigma_{\varepsilon(T_i)}}{\sigma_{\varepsilon(T_1)}}$$

$$\sigma_{\varepsilon(T_1)} \approx \sigma_{\varepsilon(T_i)}$$

$$a \approx \rho$$

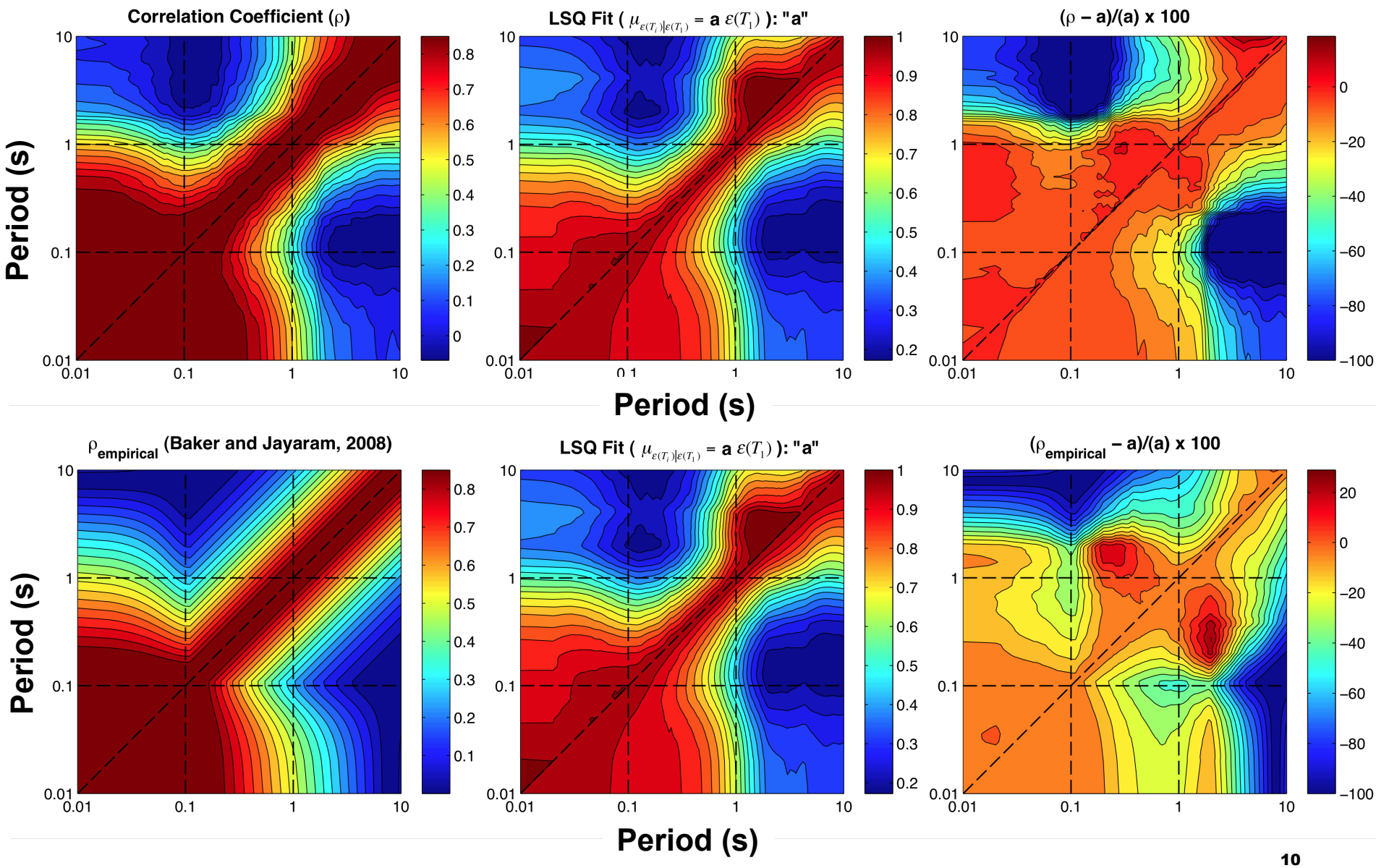


Correlation coefficient v's LSQ Fit (1 s v's 5 s)



$$\sigma_{\varepsilon(T_1)} \neq \sigma_{\varepsilon(T_i)} \qquad a \neq \rho$$

Correlation coefficient v's LSQ Fit (all T values)



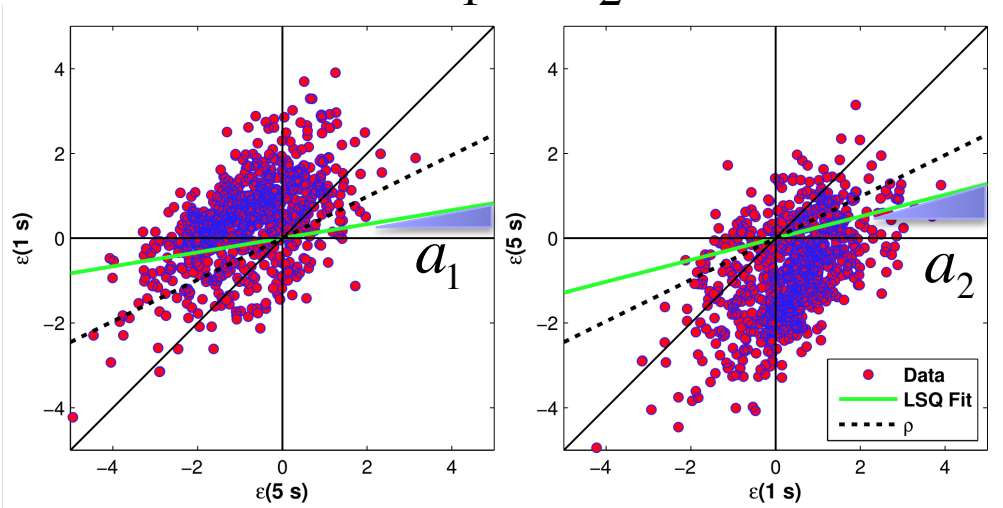
ρ matrix v's LSQ Fit slope matrix

- Correlation coefficient matrix is symmetric

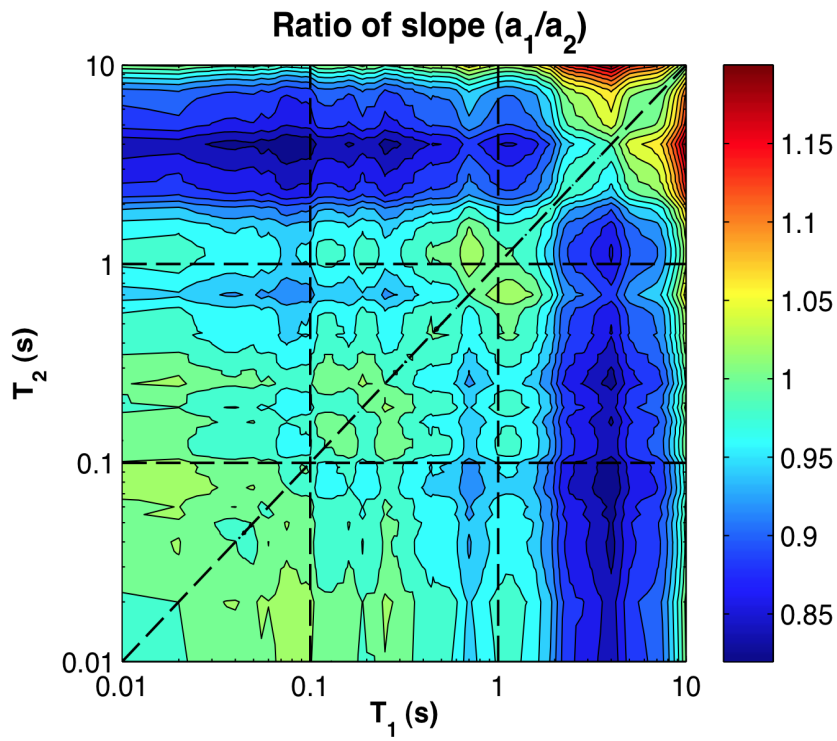
$$\rho[\varepsilon(T_1),\varepsilon(T_2)] = \rho[\varepsilon(T_2),\varepsilon(T_1)]$$

- Is LSQ Fit slope matrix symmetric?

$$a_1 \overset{?}{=} a_2$$



$$\mu_{\varepsilon(T_1)|\varepsilon(T_2)} = a_2\ \varepsilon(T_2) \quad \mu_{\varepsilon(T_2)|\varepsilon(T_1)} = a_1\ \varepsilon(T_1)$$

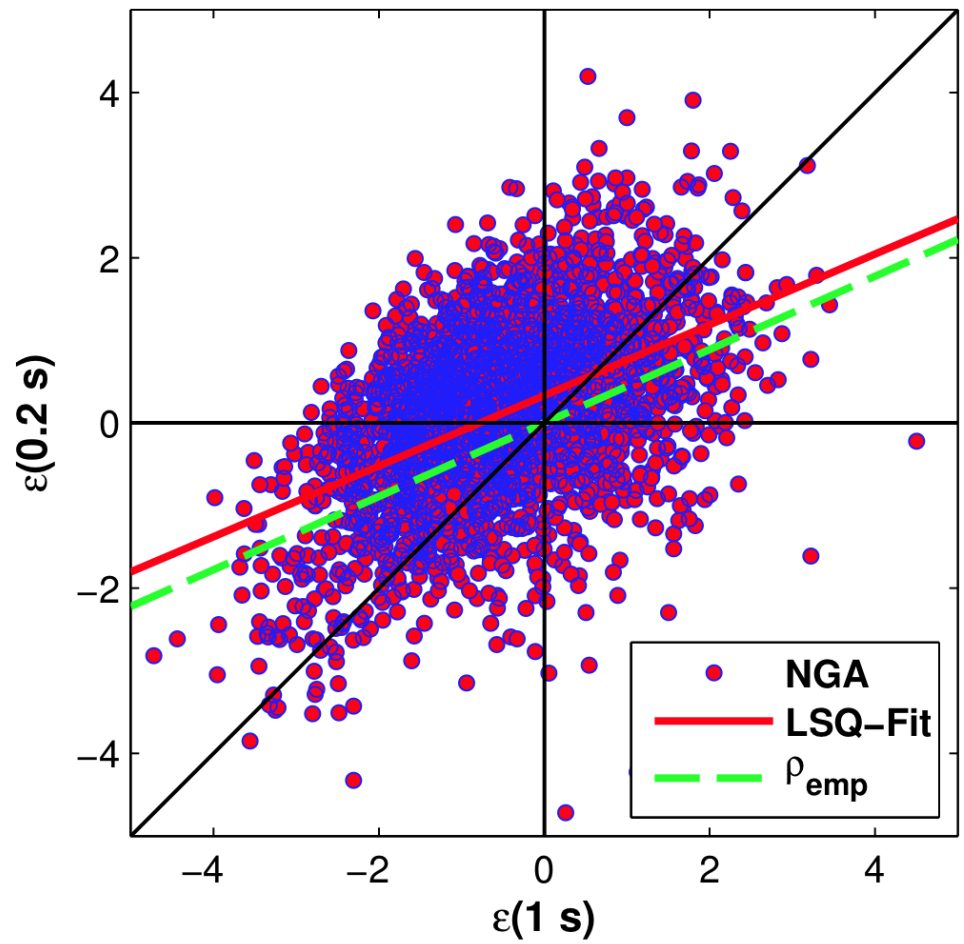


LSQ Fit with non-zero intercept

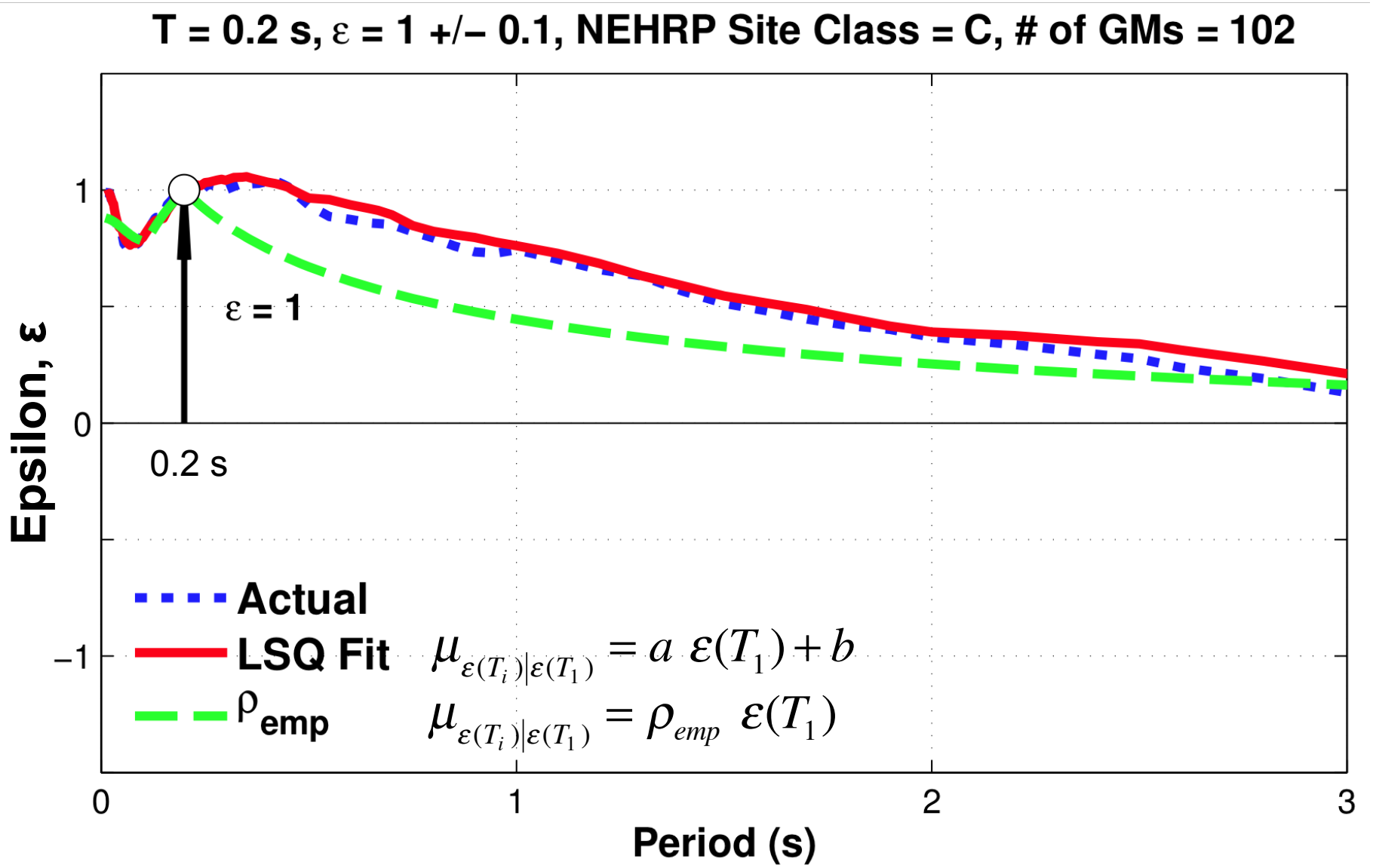
- So far, LSQ Fit is $\mu_{\varepsilon(T_i)|\varepsilon(T_1)} = a\ \varepsilon(T_1)$ (consistent with correlation coefficient, $\mu_{\varepsilon(T_i)|\varepsilon(T_1)} = \rho\ \varepsilon(T_1)$)

- Data suggests LSQ Fit to be

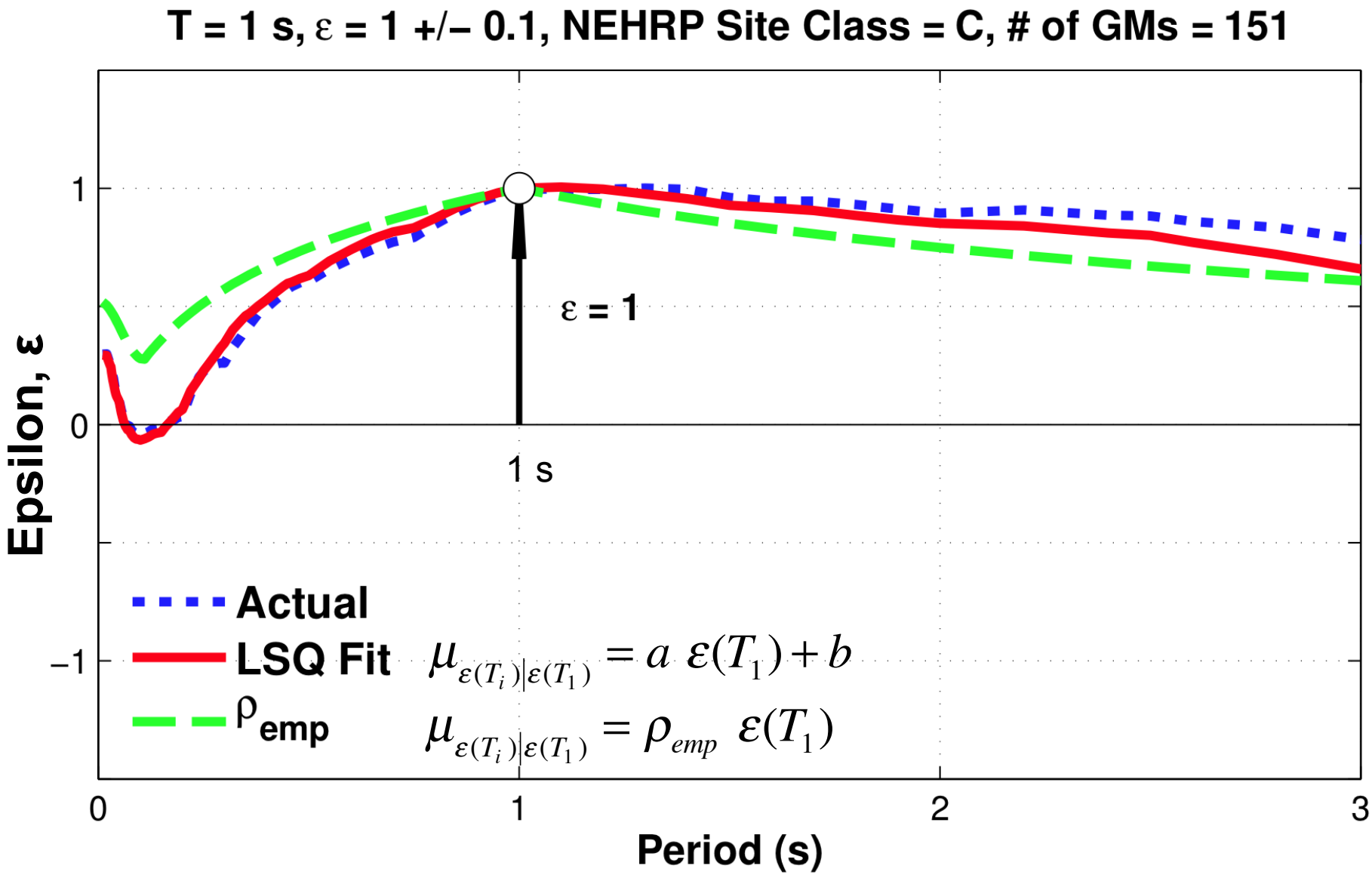
$$\mu_{\varepsilon(T_i)|\varepsilon(T_1)} = a\ \varepsilon(T_1) + b$$



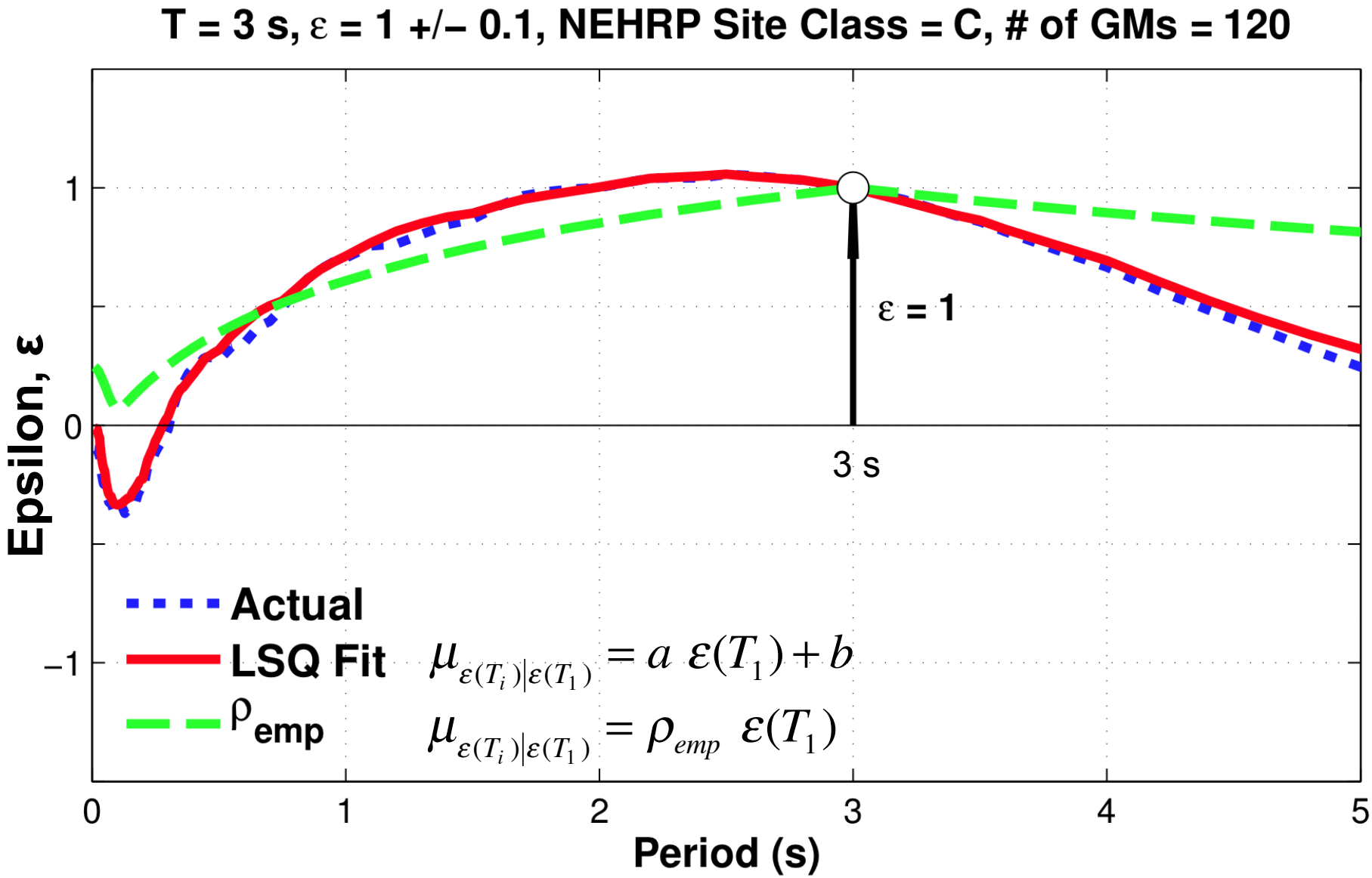
Predicted Epsilons v's "Actual" Epsilons



Predicted Epsilons v's "Actual" Epsilons

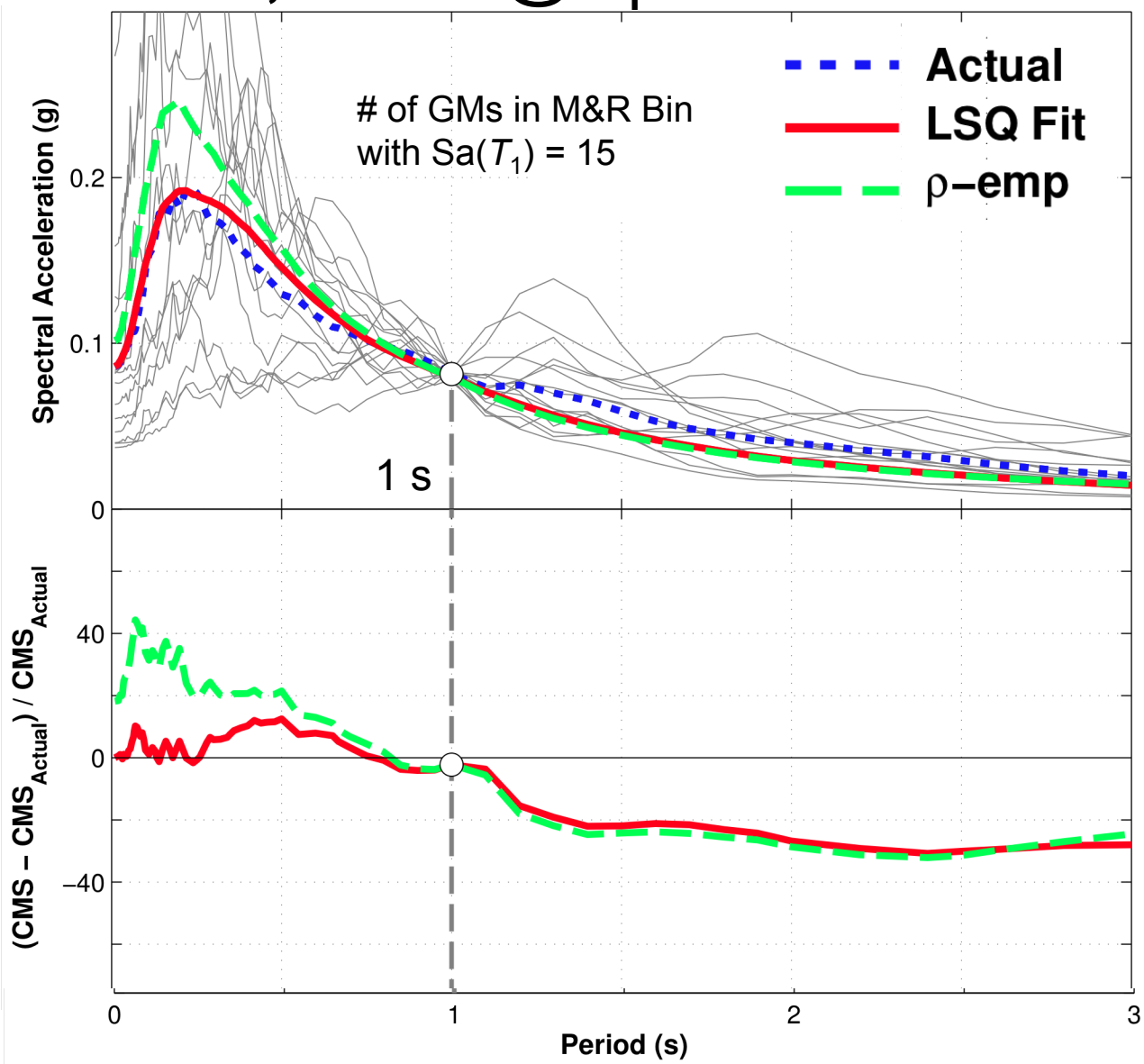


Predicted Epsilons v's "Actual" Epsilons

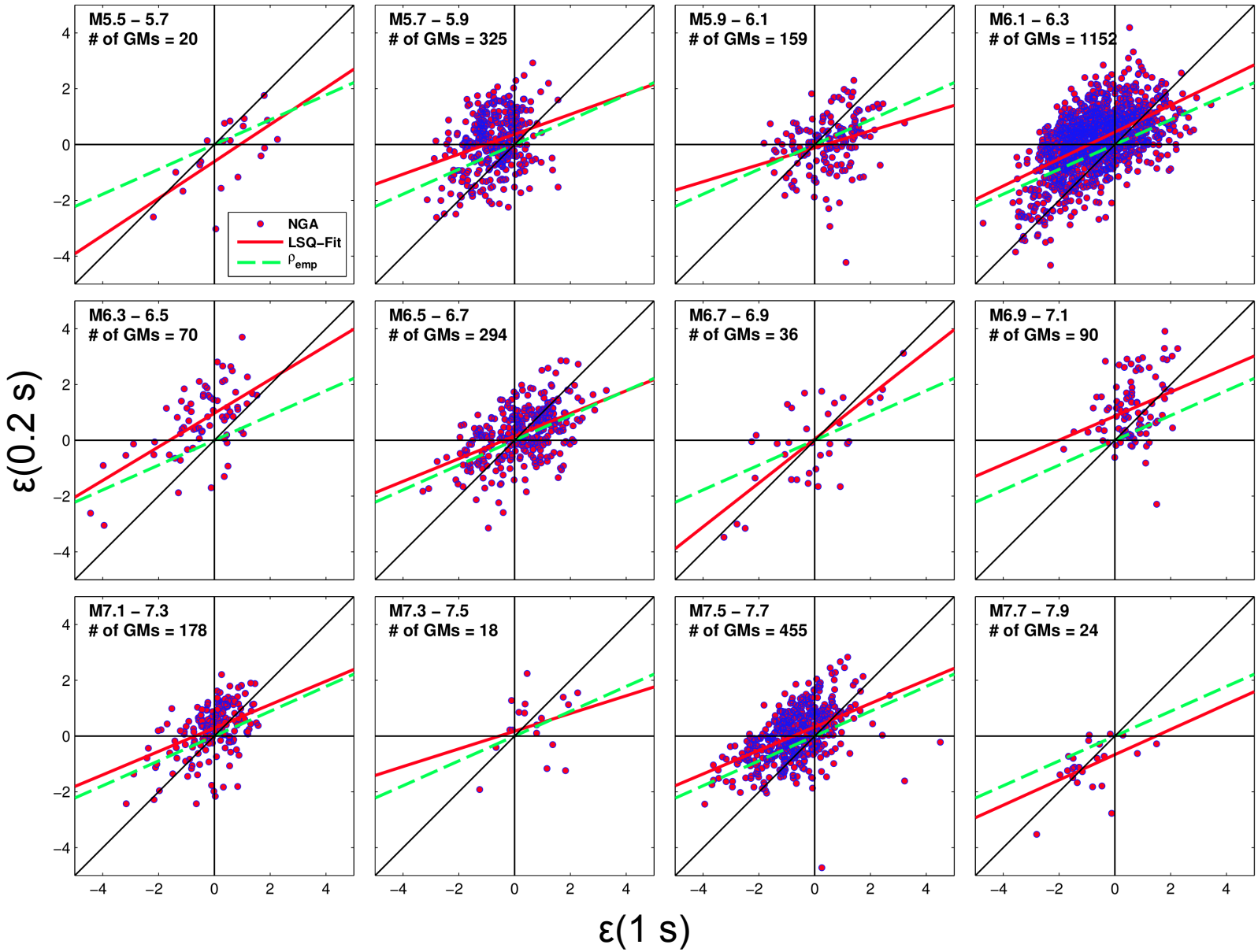


"Actual" v's Approximate CMS

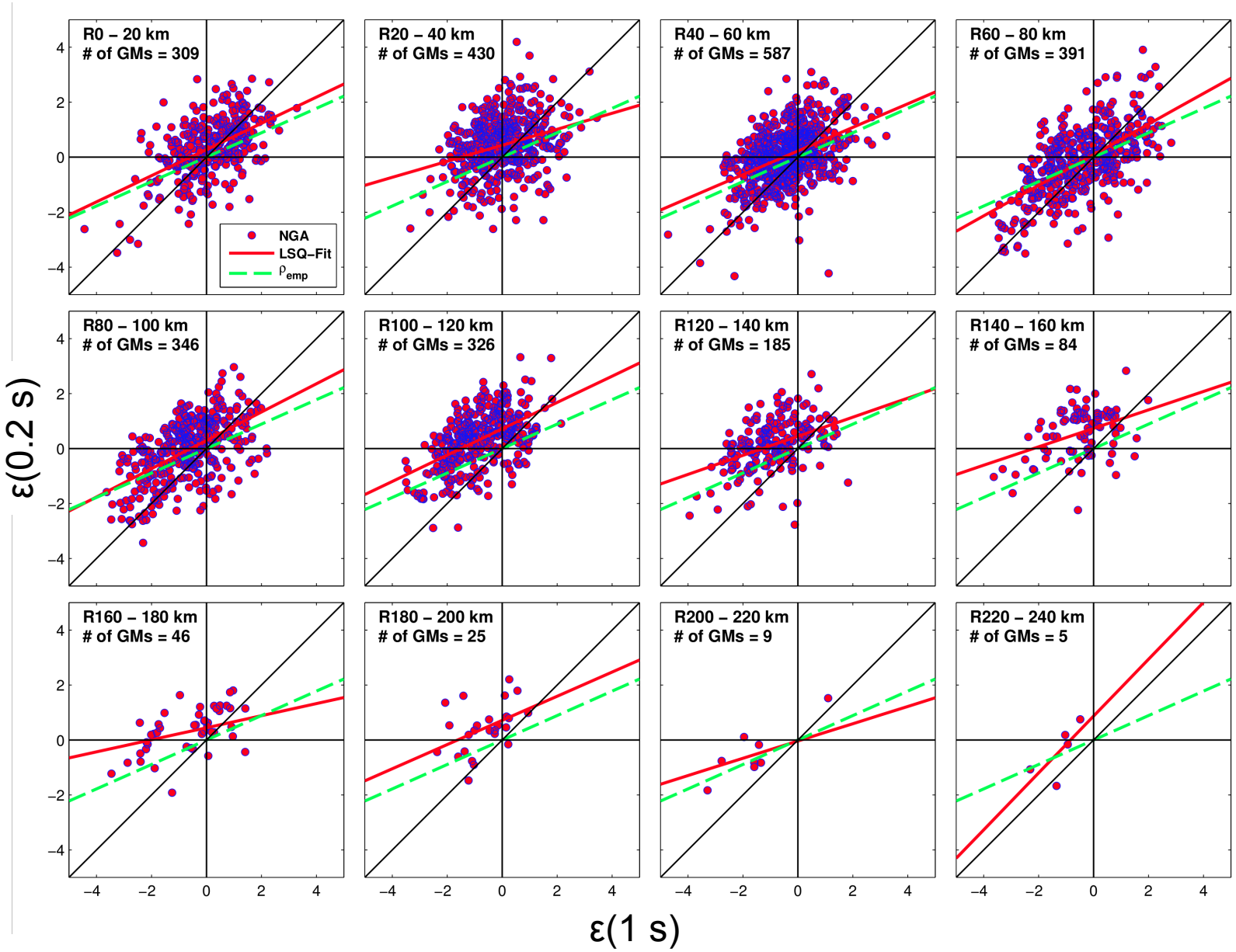
M6.3, R = 30 km, ε = 0.6 @ T_1 = 1 s



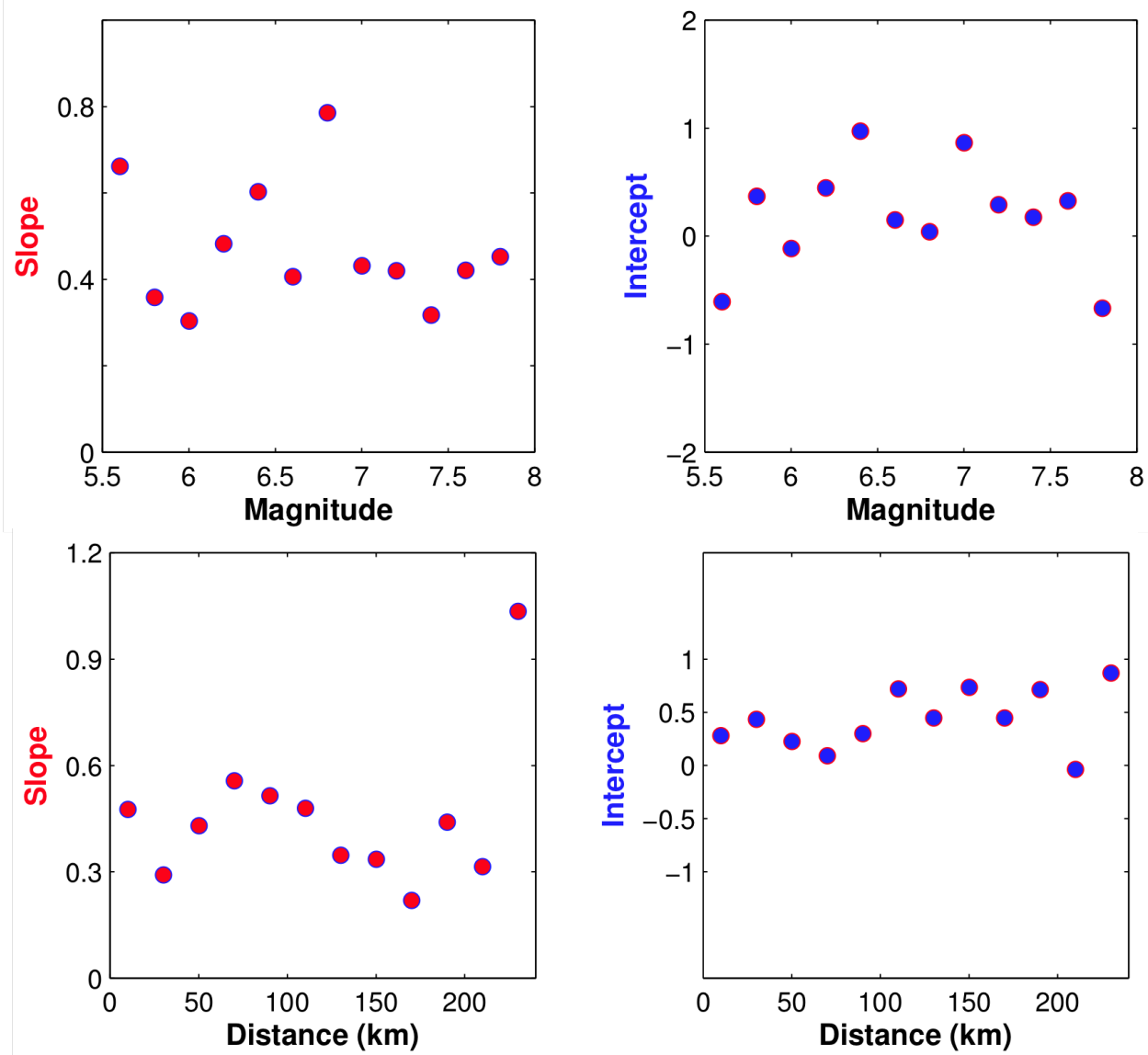
Magnitude Dependence: ε(0.2 s) v's ε(1 s)



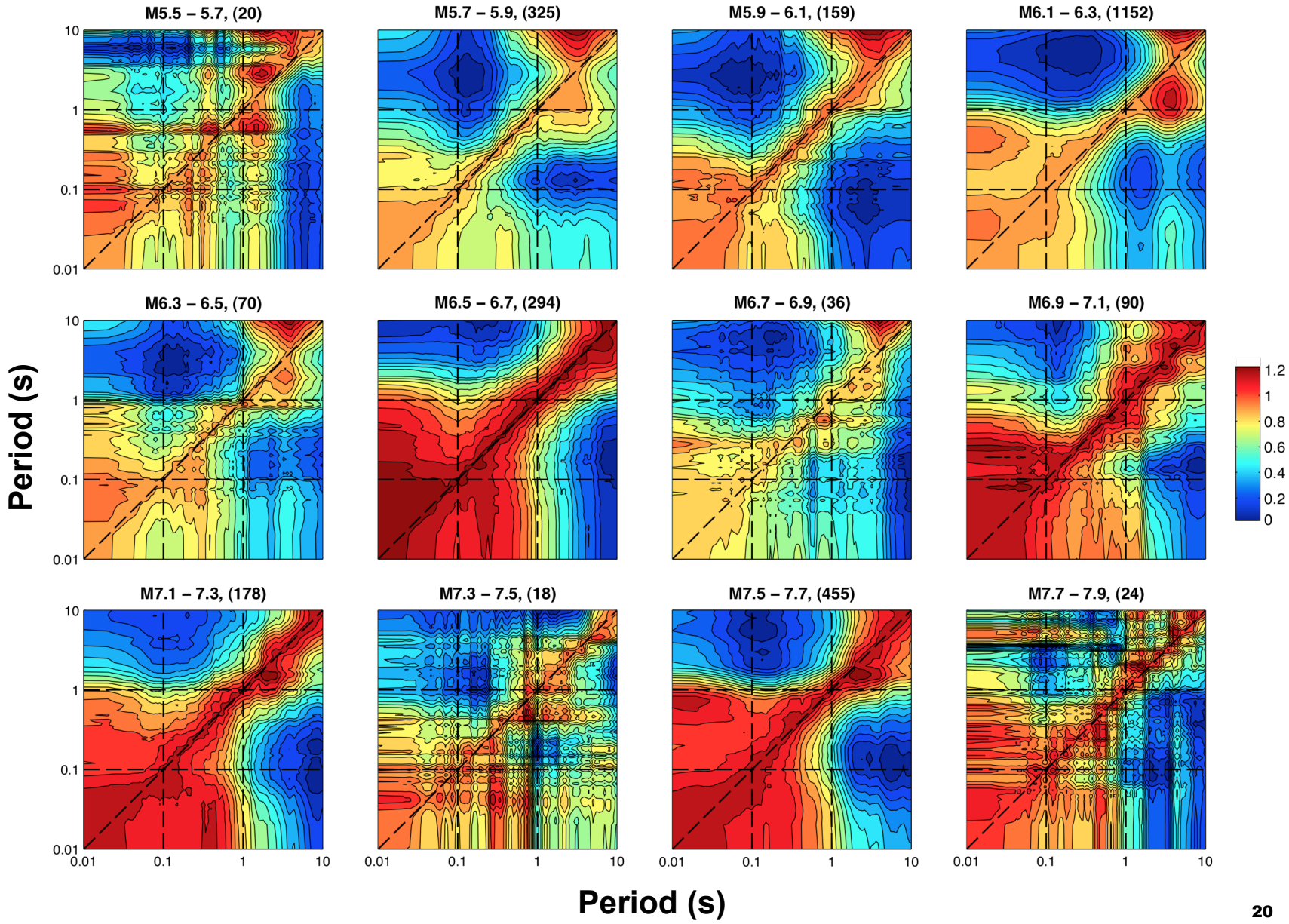
Distance Dependence: ε(0.2 s) v's ε(1 s)



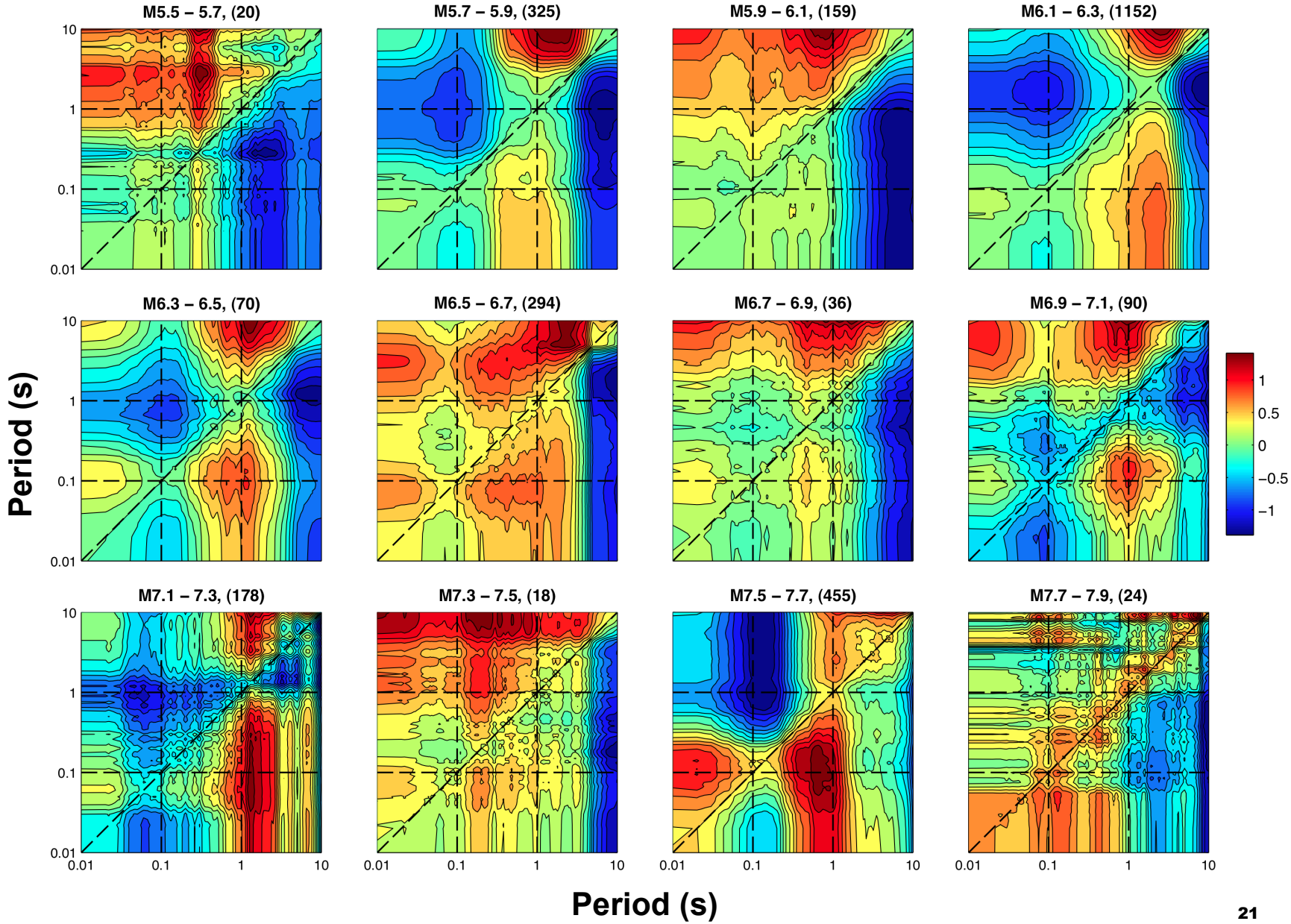
M&R Dependence in $\mu_{\varepsilon(T_i)|\varepsilon(T_1)} = \mathbf{a}\,\varepsilon(T_1) + \mathbf{b}$ (0.2 s v's 1 s)



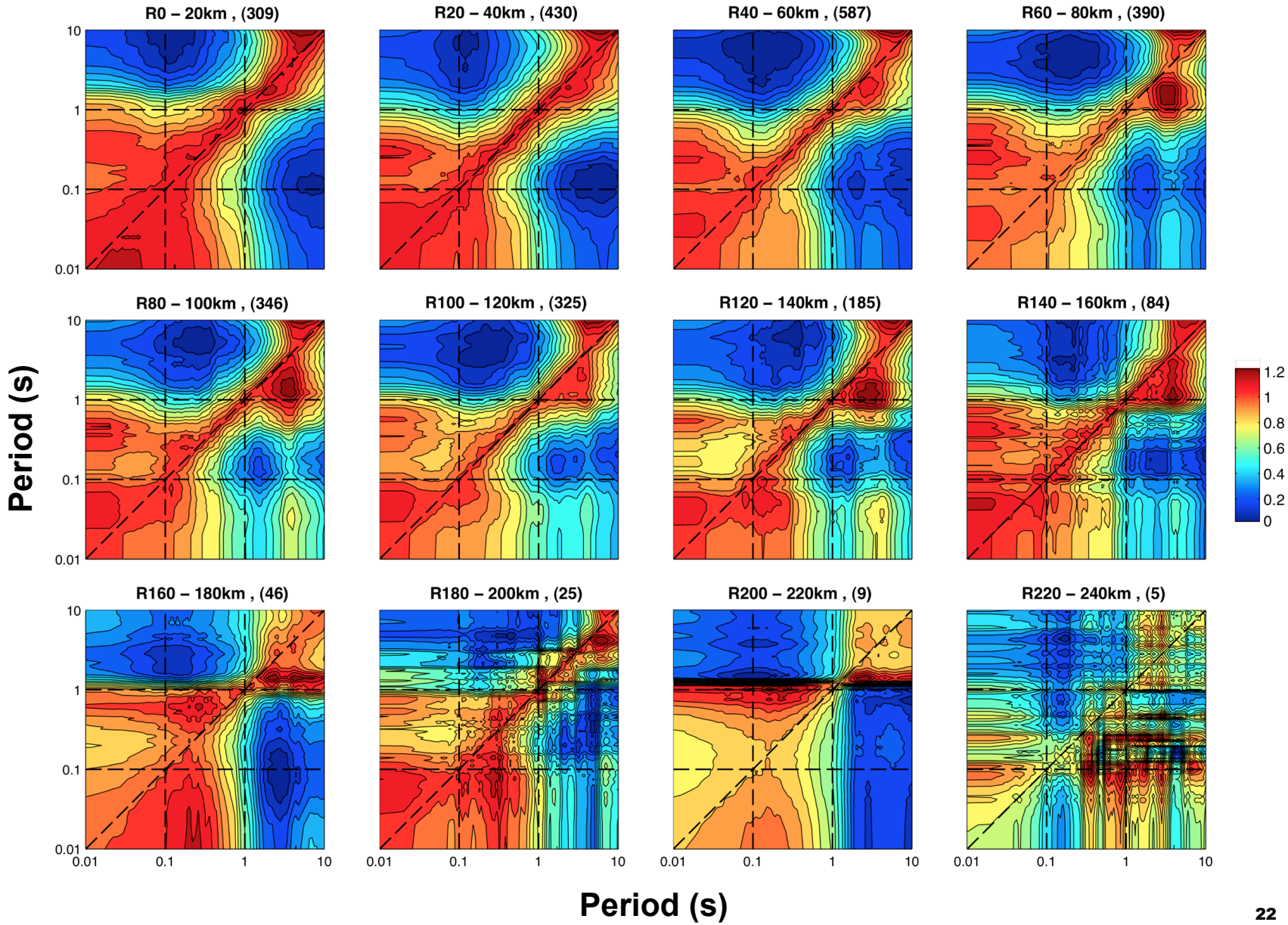
Magnitude Dependence: Slope "a" (All T values)



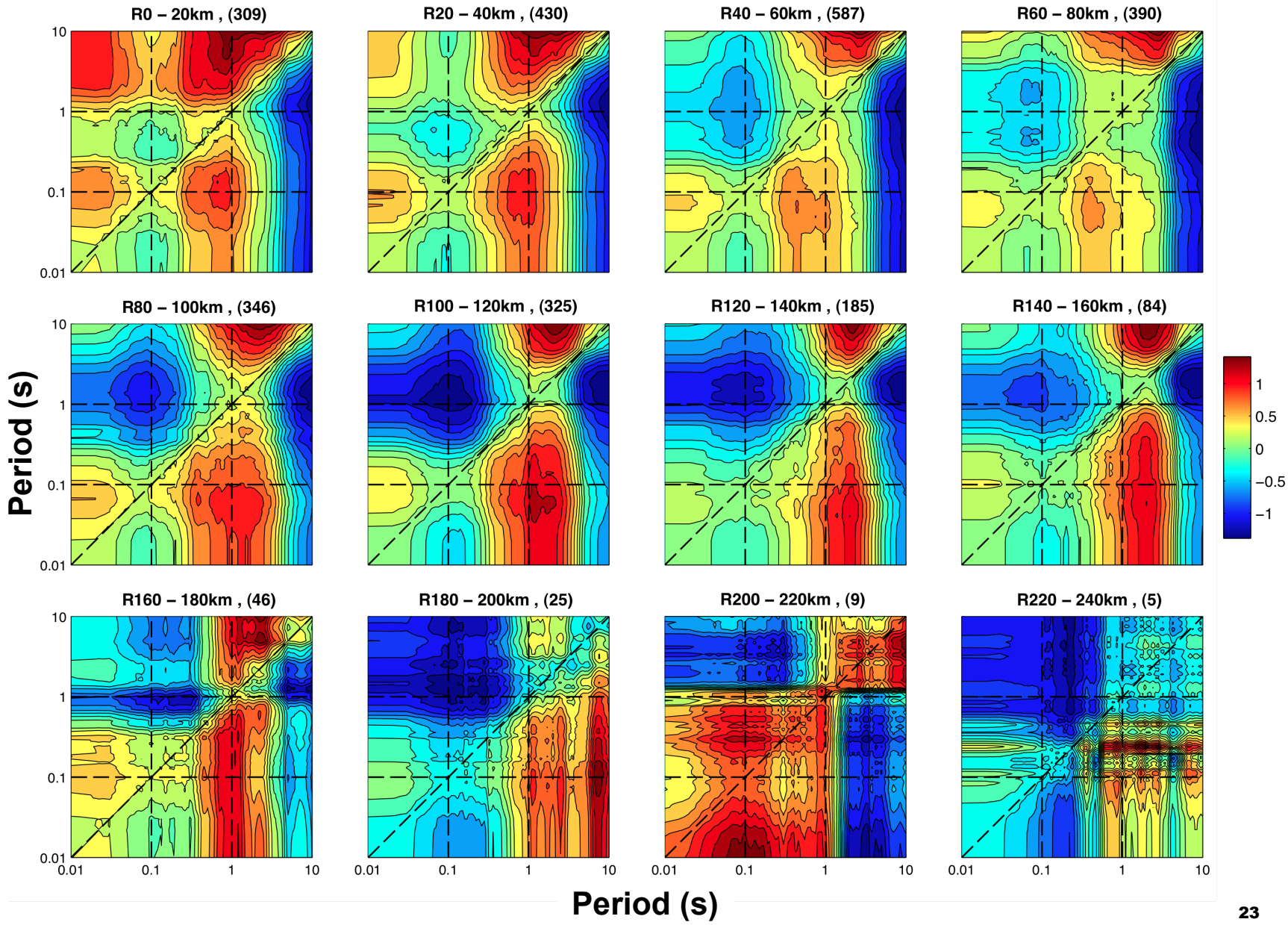
Magnitude Dependence: Intercept "b" (All T values)



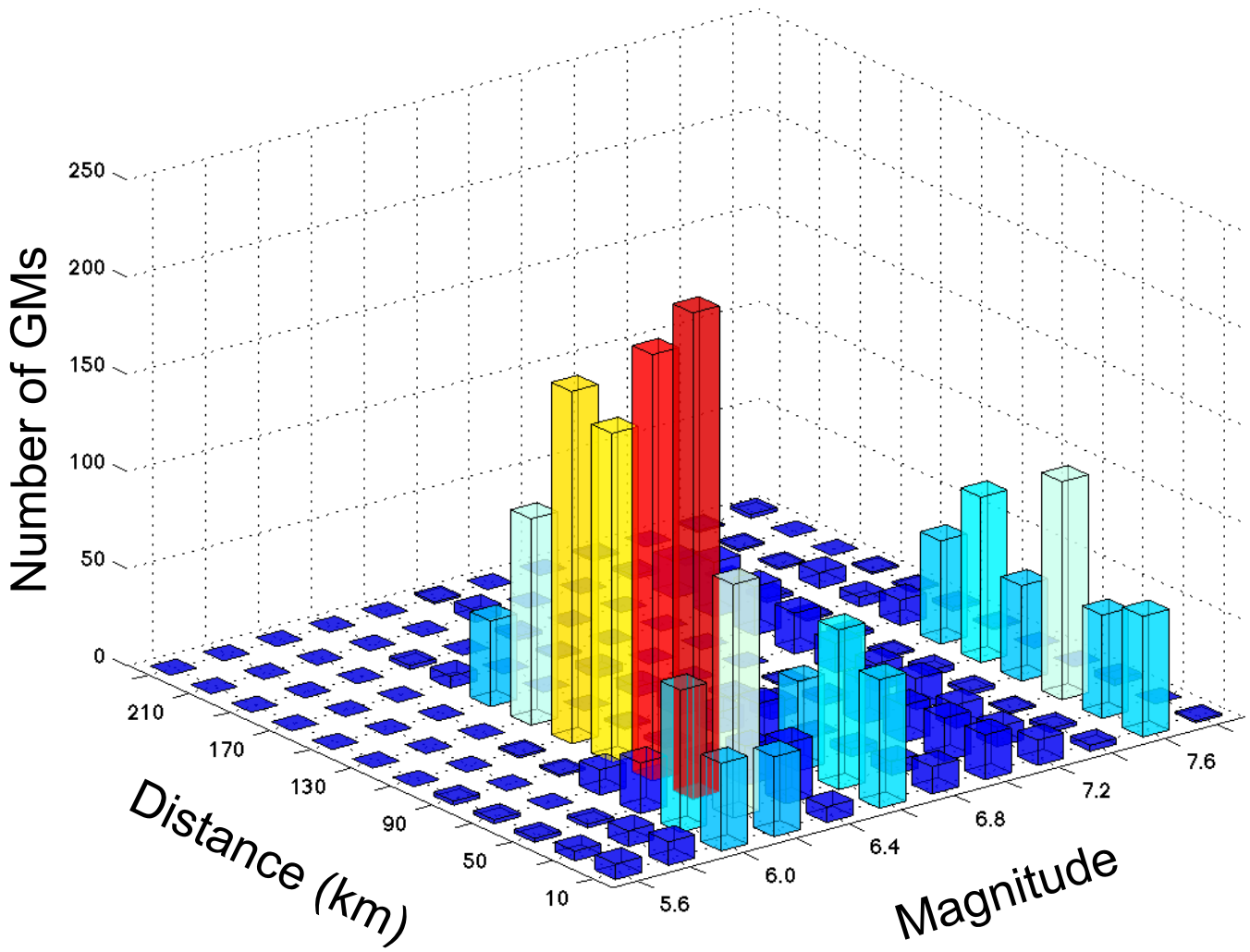
Distance Dependence: Slope "a" (All T values)



Distance Dependence: Intercept "b" (All T values)



Number of Records in NGA Database for M&R Bins



CMS based on LSQ Fit for M&R Bin

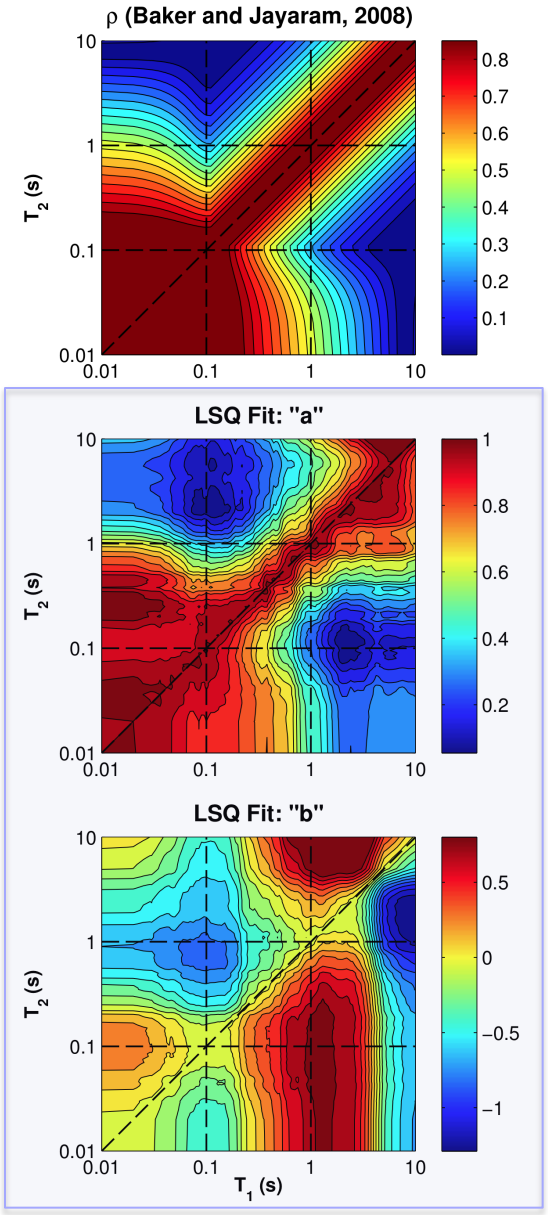
- Compile bin of GMs based on M&R from de-aggregation
- Compute epsilons for all *T* using a GMPE
- LSQ-Fit to epsilon data

$$\mu_{\varepsilon(T_i)|\varepsilon(T_1)} = a\ \varepsilon(T_1) + b$$

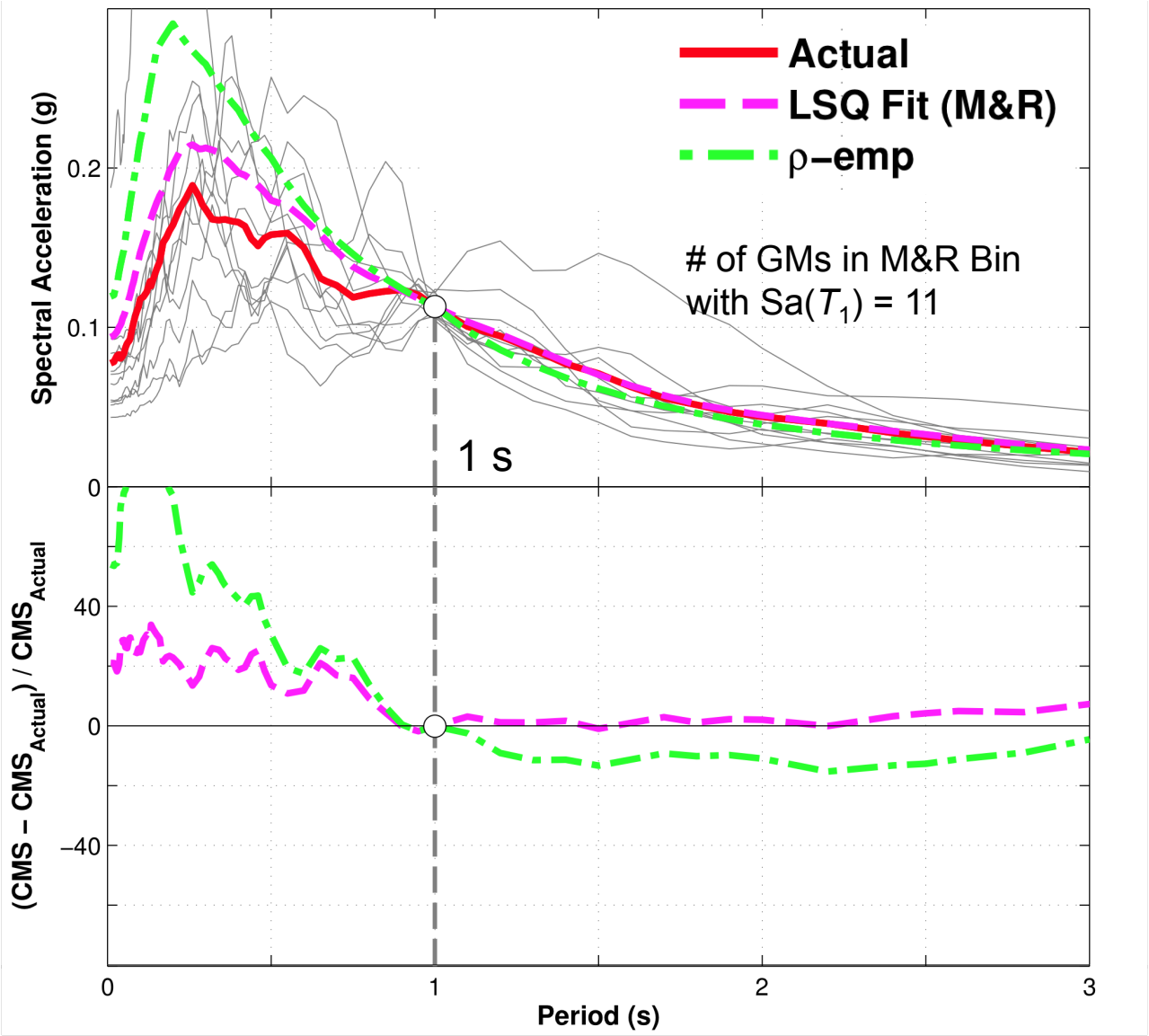
- Construct CMS

$$\mu_{lnSa(T_i)|lnSa(T_1)} = \mu_{lnSa}(M, R, Vs30, T_i) + \mu_{\varepsilon(T_i)|\varepsilon(T_1)}\ \sigma_{lnSa(T_i)}$$

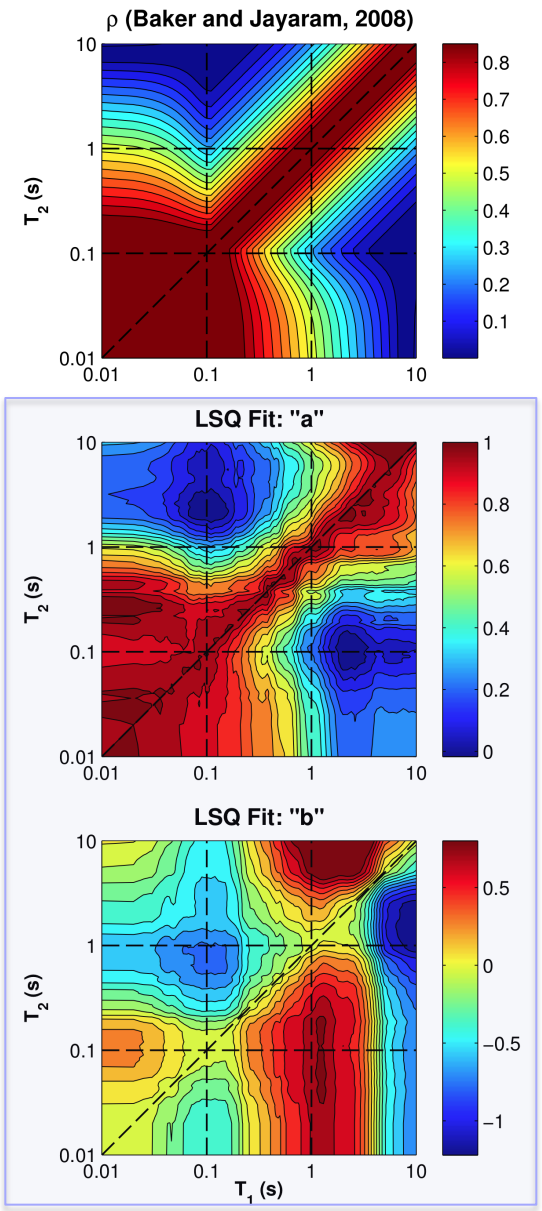
M6.4, R = 30 km, NEHRP-C, ε = 1, T =1 s



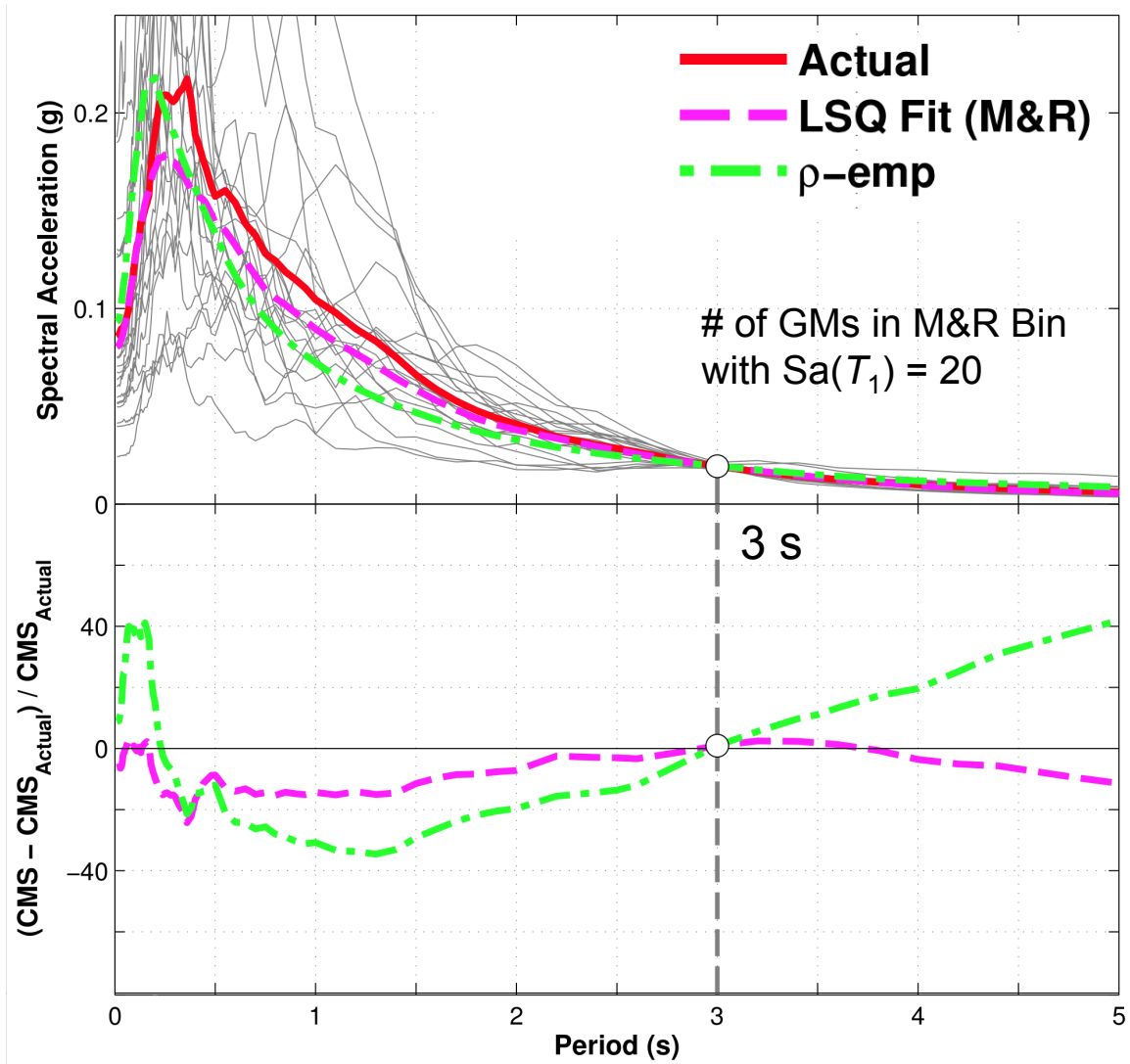
# of GMs in M&R Bin = 166



M6.2, R = 30 km, NEHRP-C, ε = 1, T = 3 s

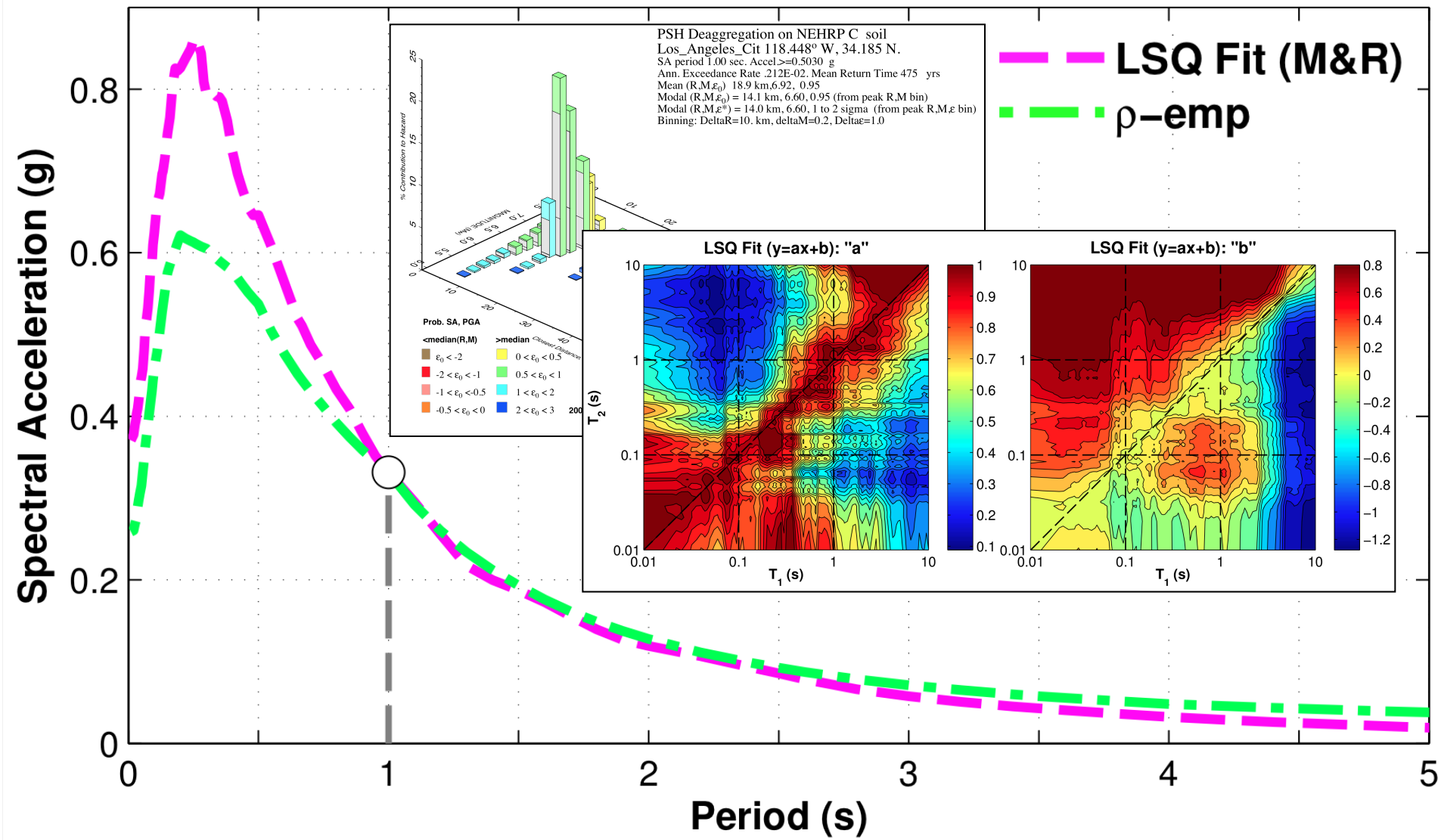


# of GMs in M&R Bin = 179



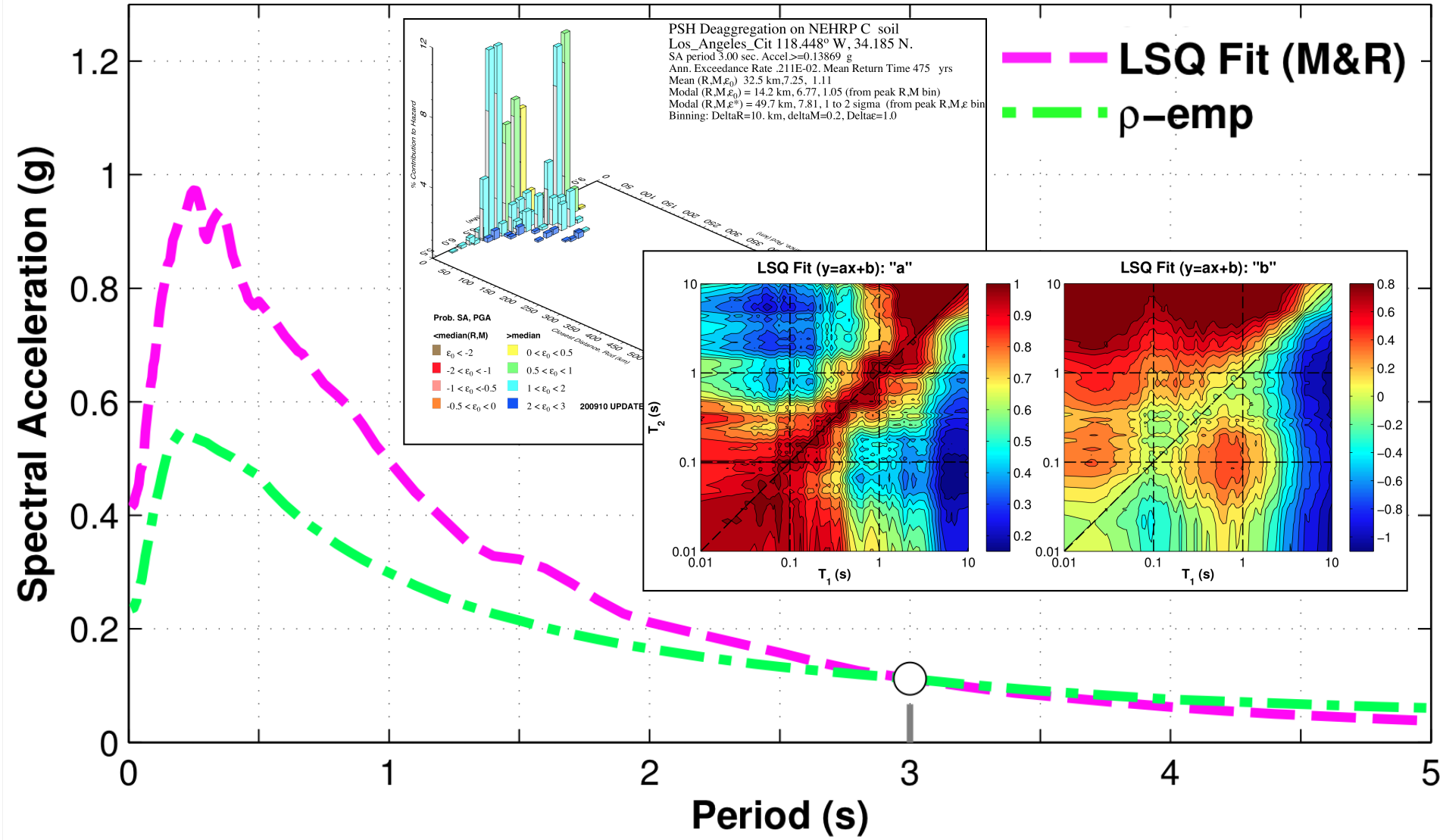
Los Angeles: M6.6, R = 14 km, ε = 0.95, T = 1 s

of GMs in M&R Bin = 38



Los Angeles: M6.8, R = 14 km, ε = 1, T = 3 s

of GMs in M&R Bin = 50



Conclusions

- Assume enough records exist for a given M, R and site condition with desired SA(T_1), then "actual" CMS is the median spectrum of these records
- Approximate CMS can be constructed using a GMPE and by relating $\varepsilon(T_1)$ and $\varepsilon(T_i)$
- CMS based on LSQ Fit for M&R bin provides better agreement with "Actual" CMS compared to other alternatives
  - LSQ Fit based on entire database
  - Correlation coefficients

Acknowledgment

We wish to thank Jack Baker for discussions and generously providing his computer codes and database

Computer Codes

- http://nsmp.wr.usgs.gov/ekalkan/CMS/index.html