Conditional Mean Spectrum: Assessment and Refinements

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Why Conditional Mean Spectrum?

- Uniform Hazard Spectrum (UHS) has been widely used as the design (or target) spectrum in practice.

- UHS envelopes the spectral amplitudes at all vibration periods with a specified exceedance probability.

- Recently proposed CMS (Baker and Cornell, 2006) is a more realistic alternative to UHS.
Objectives

- To define the “true” (or actual) CMS with given hazard conditions and $SA(T_1)$
- assess the current method to determine CMS
- develop an alternative procedure for CMS

Procedure presented:

- Based on Least Square (LSQ) Fit
- Considers magnitude (M) and distance (R) bin if there is enough data
“Actual” CMS and How to Construct It?

Assume existence of enough records with desired value of $SA(T_1)$ for specified $M$, $R$ and site condition:

$$CMS_{\text{actual}} = \text{median spectrum of such records}$$

Note: GMPE is not needed

Epsilon ($\varepsilon$) is not used
Computation of “Actual” CMS

- Magnitude
- Distance
- SA(T₁)

PSH Deaggregation on NEHRP C soil
Los_Angeles_Cit 118.448° W, 34.185 N.
SA period 1.00 sec, Avel iso/1.039 g
Ann. Exceedance Rate 212E-02, Mean Return Time 475 yrs
Mean (R.M.S.) 1.18 km/2.02, 0.95
Model (R.M.S.) 14.1 km, 6.90, 0.95 (from peak R.M.S.)
Model (R.M.S.) 14.9 km, 6.00, 1.00, 0.95 (from peak R.M.S. bin)
Binning: DeltaR=10. km, deltaM=0.2, Delta=1.0

Uniform Hazard Spectrum
CMS Actual
i^{th} Spectrum
“Approximate” CMS

- Scarcity of records for large M and small R prevents construction of actual CMS

- Approximate CMS can be constructed using a GMPE and by relating $\varepsilon(T_i)$ and $\varepsilon(T_1)$
  - Correlation analyses (Baker and Cornell, 2006)
  - Regression analyses (LSQ Fit)
Regression v’s Correlation Analyses

Correlation Coefficient

\[ \rho_{\varepsilon(T_1), \varepsilon(T_i)} = \frac{\text{cov}[\varepsilon(T_1), \varepsilon(T_i)]}{\sigma_{\varepsilon(T_1)} \sigma_{\varepsilon(T_i)}} \]

LSQ Fit

\[ \mu_{\varepsilon(T_i)|\varepsilon(T_1)} = a(\text{or } \rho) \varepsilon(T_1) \]
Correlation coefficient v’s LSQ Fit (1 s v’s 1.2 s)

\[
a_{\varepsilon(T_i),\varepsilon(T_1)} = \rho_{\varepsilon(T_1),\varepsilon(T_i)} \frac{\sigma_{\varepsilon(T_i)}}{\sigma_{\varepsilon(T_1)}}
\]

\[
\sigma_{\varepsilon(T_1)} \approx \sigma_{\varepsilon(T_i)}
\]

\[
a \approx \rho
\]
Correlation coefficient v’s LSQ Fit (1 s v’s 5 s)

\[ \sigma_{\varepsilon(T_1)} \neq \sigma_{\varepsilon(T_i)} \quad \alpha \neq \rho \]
Correlation coefficient v’s LSQ Fit (all T values)
Correlation coefficient matrix is symmetric

\[ \rho[\varepsilon(T_1), \varepsilon(T_2)] = \rho[\varepsilon(T_2), \varepsilon(T_1)] \]

Is LSQ Fit slope matrix symmetric?

\[ a_1 = a_2 \]

\[ \mu_{\varepsilon(T_1)\varepsilon(T_2)} = a_2 \varepsilon(T_2), \quad \mu_{\varepsilon(T_2)\varepsilon(T_1)} = a_1 \varepsilon(T_1) \]
LSQ Fit with non-zero intercept

- So far, LSQ Fit is $\mu_{\varepsilon(T_i)|\varepsilon(T_1)} = a \varepsilon(T_1)$ (consistent with correlation coefficient, $\mu_{\varepsilon(T_i)|\varepsilon(T_1)} = \rho \varepsilon(T_1)$)

- Data suggests LSQ Fit to be $\mu_{\varepsilon(T_i)|\varepsilon(T_1)} = a \varepsilon(T_1) + b$
Predicted Epsilons v’s “Actual” Epsilons

$T = 0.2 \text{ s, } \varepsilon = 1 \pm 0.1$, NEHRP Site Class = C, # of GMs = 102

\[
\varepsilon = 1
\]

\[
\mu_{\varepsilon(T_i)|\varepsilon(T_1)} = a \cdot \varepsilon(T_1) + b
\]

\[
\mu_{\varepsilon(T_i)|\varepsilon(T_1)} = \rho_{\text{emp}} \cdot \varepsilon(T_1)
\]
Predicted Epsilons v’s “Actual” Epsilons

$T = 1 \text{ s}, \varepsilon = 1 \pm 0.1, \text{ NEHRP Site Class} = C, \# \text{ of GMs} = 151$
$T = 3\ \text{s, } \varepsilon = 1 \pm 0.1, \ \text{NEHRP Site Class = C, } \# \text{ of GMs} = 120$

Predicted Epsilons v's "Actual" Epsilons

$\mu_{\varepsilon(T_i) | \varepsilon(T_1)} = a \varepsilon(T_1) + b$

$\mu_{\varepsilon(T_i) | \varepsilon(T_1)} = \rho_{\text{emp}} \varepsilon(T_1)$
“Actual” v’s Approximate CMS
M6.3, R = 30 km, $\varepsilon = 0.6$ @ $T_1 = 1$ s

# of GMs in M&R Bin with $\text{Sa}(T_1) = 15$
Magnitude Dependence: $\varepsilon(0.2\ s)$ v’s $\varepsilon(1\ s)$
Distance Dependence: $\varepsilon(0.2\text{ s})$ v’s $\varepsilon(1\text{ s})$
M&R Dependence in $\mu_{\varepsilon(T_1)\varepsilon(T_1)} = a \varepsilon(T_1) + b$

(0.2 s v’s 1 s)
Magnitude Dependence: Slope “a” (All T values)
Magnitude Dependence: Intercept “b” (All T values)
Distance Dependence: Slope “a” (All T values)
Distance Dependence: Intercept “b” (All T values)
Number of Records in NGA Database for M&R Bins

Number of GMs

Distance (km)

Magnitude

Number of GMs
CMS based on LSQ Fit for M&R Bin

- Compile bin of GMs based on M&R from de-aggregation
- Compute epsilons for all $T$ using a GMPE
- LSQ-Fit to epsilon data

$$\mu_{\varepsilon(T_i)|\varepsilon(T_1)} = a \varepsilon(T_1) + b$$

- Construct CMS

$$\mu_{\lnSa(T_i)|\lnSa(T_1)} = \mu_{\lnSa(M,R,Vs30,T_i)} + \mu_{\varepsilon(T_i)|\varepsilon(T_1)} \sigma_{\lnSa(T_i)}$$
M6.4, R = 30 km, NEHRP-C, $\varepsilon = 1$, $T = 1$ s

# of GMs in M&R Bin = 166

# of GMs in M&R Bin with $\text{Sa}(T_1) = 11$
M6.2, R = 30 km, NEHRP-C, $\varepsilon = 1$, $T = 3$ s

# of GMs in M&R Bin = 179

# of GMs in M&R Bin with $Sa(T_1) = 20$

3 s
Los Angeles: M6.6, R = 14 km, $\varepsilon = 0.95$, $T = 1$ s

# of GMs in M&R Bin = 38
Los Angeles: M6.8, R = 14 km, \( \varepsilon = 1 \), \( T = 3 \) s

# of GMs in M&R Bin = 50
Conclusions

- Assume enough records exist for a given M, R and site condition with desired SA($T_1$), then “actual” CMS is the median spectrum of these records.

- Approximate CMS can be constructed using a GMPE and by relating $\varepsilon(T_1)$ and $\varepsilon(T_i)$.

- CMS based on LSQ Fit for M&R bin provides better agreement with “Actual” CMS compared to other alternatives.
  - LSQ Fit based on entire database
  - Correlation coefficients
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Computer Codes