



# FAST NONLINEAR RESPONSE HISTORY ANALYSIS

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## Abstract

Nonlinear response history analysis (RHA) has been a powerful tool in performance-based seismic engineering for validating proposed design of new or performance assessment of existing structures. In this approach, the seismic demands are determined by nonlinear RHAs of the structure excited by several ground motion acceleration records. When it is applied to structural systems with a large number of degrees of freedom such as three-dimensional models of tall buildings, bridges, or dams, the analyses can be computationally challenging and time consuming. The prolonged computing times become even more prominent in parametric studies or in incremental dynamic analyses where a computer model of the structure is subjected to a series of nonlinear RHAs by systematically increasing the intensity of input excitation. In order to reduce the computation time, this study proposes a practical method whereby leading and trailing weak signals in the acceleration record are trimmed, and remaining record is downsampled. The proposed method preserves significant frequency characteristics of the original record including its *S*-phase. The parameter to identify leading and trailing portions of the record to be trimmed is the maximum roof displacement calculated by implementing the uncoupled modal response history analysis. This parameter is selected because it represents the characteristics of both the ground motion and structural response with the goal of obtaining a highly efficient RHA without significant error. Test results based on three-dimensional computer models of idealized 5, 10, 15 and 20-story reinforced concrete structures demonstrate that the proposed method is not only viable, but also capable of controlling discrepancies in estimates of engineering demand parameters (EDPs) such as peak roof displacement. Performance of the proposed method is evaluated in terms of a goodness of fit test, comparing peak roof displacements obtained from the trimmed and downsampled records under uni-directional excitations with those from the original records.

*Keywords:* Nonlinear response history analysis; Finite element modeling; Computation time; Accelerograms; Downsampling.

## 1. Introduction

As performance-based seismic design considerations have become pre-requisite for controlling the level of structural and non-structural damage during an earthquake, the use of nonlinear response history analysis (RHA) has gained utmost importance. This rigorous analysis method requires, as input, a suite of ground motion acceleration records. For three-dimensional (3D) RHAs, pairs of seven records are often used per Chapter 16 of ASCE/SEI 7-10 [1]—a standard document for nonlinear analyses of tall and special buildings (e.g., base-isolated) according to both California Building Code [2] and International Building Code [3]. The number of required ground motions increases to eleven pairs in the upcoming ASCE/SEI 7-16 [4]. When it is applied to structural systems with a large number of degrees of freedom such as 3D computer models of tall buildings or complex structures (e.g., dams and bridges), the nonlinear RHAs can be computationally challenging and time consuming. For ground-motion records with high sampling rates (e.g. 200 or more samples-per-second) with long durations (associated with large magnitude earthquakes), the analysis time can be particularly time consuming. The prolonged computing times become even more prominent in parametric studies or in incremental dynamic analyses [5] where a computer model of the structure is subjected to a series of nonlinear RHAs by systematically increasing the intensity of input excitations.

One approach to achieve computational efficiency is downsampling acceleration time histories, which reduces the number of steps in the analysis. For instance, a superposition of a relatively small number of pulses may be used to represent ground-motion records; such representation is obtained by the expansion of velocity in orthogonal wavelet series using the fast wavelet transform, and approximation by only the largest energy terms in the series [6]. In a more recent approach, filtering and downsampling techniques are used to generate the downsampled record with a corresponding time step that is based on the frequency response function of the structure representing characteristics of a given record and structural system [7].

With the goal of obtaining a highly efficient RHA without significant error, this study proposes a new approach by appropriately trimming beginning and end of acceleration record and downsampling the remaining record while preserving significant frequency characteristics of the original record including its *S*-phase. The parameter to identify leading and trailing signals to be trimmed is the maximum roof displacement calculated by implementing the uncoupled modal response history analysis (UMRHA) [8-12]. This parameter is found to be superior to other candidates such as Arias intensity [13] and yield base shear because it represents the characteristics of both the ground motion and structural response.

Test results based on 3D computer models of idealized 5, 10, 15 and 20-story reinforced concrete structures demonstrate that the proposed method is not only viable, but also capable of controlling errors in estimates of peak roof displacement. The goodness of fit test was used to compare the estimates of this engineering demand parameter (EDP) from the trimmed and downsampled records against those from the original records. This study is based on uni-directional far-field records. Among other EDPs studied such as inter-story drift ratio, floor acceleration etc., only the peak roof displacement results are presented here due to page limitation. The extension of the method to other EDPs and bi-direction excitation are reported in [14].

## 2. Ground Motions Selected

For this study, 30 ground motion acceleration records (listed in Table 1) were compiled from seven shallow crustal earthquakes with moment magnitude  $6.7 \pm 0.2$ , at Joyner-Boore distances ranging from 20 to 30 km, and with National Earthquake Engineering Hazard Reduction Program (NEHRP) site classification C or D (very dense soil and soft rock or stiff soil) from the Next Generation of Attenuation ground-motion database (<http://ngawest2.berkeley.edu/>, last accessed May 2016). The median spectrum of the 30 far-field ground motion records is taken as the target spectrum for spectrum matching [15-16]. The difference between the target spectrum and the spectrum of each record is used as a selection criterion. If the record has a spectral shape very different than the target spectrum, it is discarded. The final set of seven records selected has spectral shapes that match closely with the target spectrum before the spectrum matching. The number of selected records was limited to seven because previous research shows that a minimum of seven records is sufficient for unbiased estimates of EDPs from nonlinear RHAs [17-18]. Fig. 1 depicts the 5%-damped median response spectra for *x* and *y* components of the selected records and the median spectra of 30 initial records.

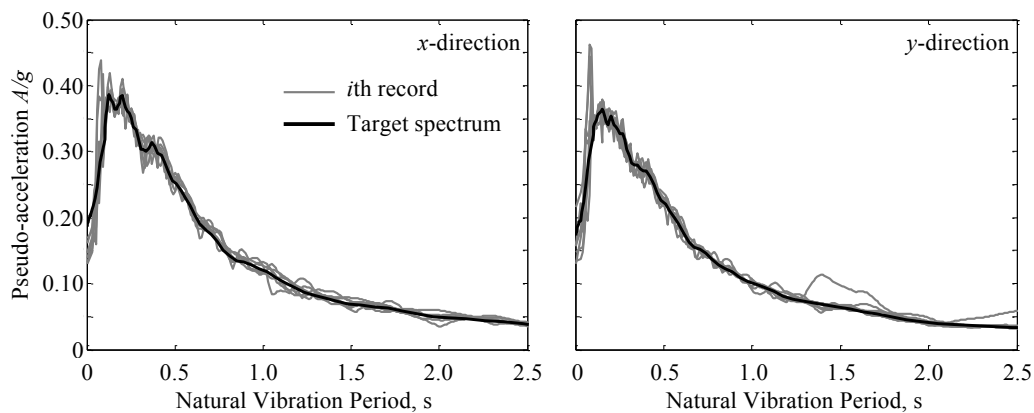


Fig. 1 – Geometric-mean pseudo-acceleration response spectra of 30 records at 5% damping in *x*- and *y*-directions used for seismic design of computer models; also shown are response spectra of seven spectral-matched records used for testing the proposed methodology.

Table 1 – List of ground motions [NEHRP: National Earthquake Hazard Reduction Program]

No.	Earthquake	Year	Station	Moment magnitude	Joyner-Boore distance (km)	NEHRP Soil class
1	San Fernando	1971	LA - Hollywood Stor FF	6.61	22.77	D
2	San Fernando	1971	Santa Felita Dam (Outlet)	6.61	24.69	C
3	Imperial Valley	1979	Calipatria Fire Station	6.53	23.17	D
4	Imperial Valley	1979	Delta	6.53	22.03	D
5	Imperial Valley	1979	El Centro Array #1	6.53	19.76	D
6	Imperial Valley	1979	El Centro Array #13	6.53	21.98	D
7	Imperial Valley	1979	Superstition Mtn Camera	6.53	24.61	C
8	Irpinia, Italy	1980	Brienza	6.90	22.54	C
9	Superstition Hills	1987	Wildlife Liquef. Array	6.54	23.80	D
10	Loma Prieta	1989	Agnews State Hospital	6.93	24.27	D
11	Loma Prieta	1989	Anderson Dam (Downst)	6.93	19.90	C
12	Loma Prieta	1989	Anderson Dam (L Abut)	6.93	19.90	C
13	Loma Prieta	1989	Coyote Lake Dam (Downst)	6.93	20.44	D
14	Loma Prieta	1989	Coyote Lake Dam (SW Abut)	6.93	19.97	C
15	Loma Prieta	1989	Gilroy Array #7	6.93	22.36	D
16	Loma Prieta	1989	Hollister - SAGO Vault	6.93	29.54	C
17	Northridge	1994	Castaic - Old Ridge Route	6.69	20.10	C
18	Northridge	1994	Glendale - Las Palmas	6.69	21.64	C
19	Northridge	1994	LA - Baldwin Hills	6.69	23.51	D
20	Northridge	1994	LA - Centinela St	6.69	20.36	D
21	Northridge	1994	LA - Cypress Ave	6.69	28.98	C
22	Northridge	1994	LA - Fletcher Dr	6.69	25.66	C
23	Northridge	1994	LA - N Westmoreland	6.69	23.40	D
24	Northridge	1994	LA - Pico & Sentous	6.69	27.82	D
25	Kobe, Japan	1995	Abeno	6.90	24.85	D
26	Kobe, Japan	1995	Kakogawa	6.90	22.50	D
27	Kobe, Japan	1995	Morigawachi	6.90	24.78	D
28	Kobe, Japan	1995	OSAJ	6.90	21.35	D
29	Kobe, Japan	1995	Sakai	6.90	28.08	D
30	Kobe, Japan	1995	Yae	6.90	27.77	D

### 3. Structural Systems

Considered in this study are 5, 10, 15 and 20-story buildings with a similar plan and floor weights. Each structure has a span length  $l_x$  and  $l_y$  of 25 m (82.02 ft.), a story height of 3 m (9.84 ft.), and uniformly distributed floor load of 10 kN/m<sup>2</sup> (208.85 psf.).

The idealized structure with three main degree of freedoms in three-dimensions (3D) (Fig. 2) was described as a shear model containing two vertical elements in each horizontal direction (Fig. 3a). A trilinear constitutive model was used to define these elements (Fig. 3b). The structural system has a constant initial stiffness  $k_1$  over its height.  $k_1$  was adjusted to achieve a prescribed fundamental period  $T_1$  to ensure that the fundamental vibration modes were sufficiently separated so that the complete quadratic combination (CQC) rule could be used.  $T_1$  is defined as a function of the number of stories per equations 8-7 in Chapter 12 of ASCE/SEI

7-10 [1] using the parameters for concrete moment-resisting frames. The earthquake design forces  $V_y$  were determined by bi-directional linear response spectrum analysis of the building with the spectrum equal to the target spectrum shown in Fig. 1. 5% damping ratio was assigned to all modes of vibration. The maximum shear force in the elastic range was estimated by dividing the earthquake design forces  $V_y$  by a response modification coefficient  $R$  equal to 3, 5, 7 and a value that leads to linear elastic design. The over-strength factor (2.0) and the amplification factor of the yield drift  $\Delta_y$  (3.5) were selected based on pushover curves for the first mode of several structures [19]. Considering the different values for  $T_n$  and  $R$ , sixteen structures were characterized. A summary of the structural parameters is listed in Table 2.

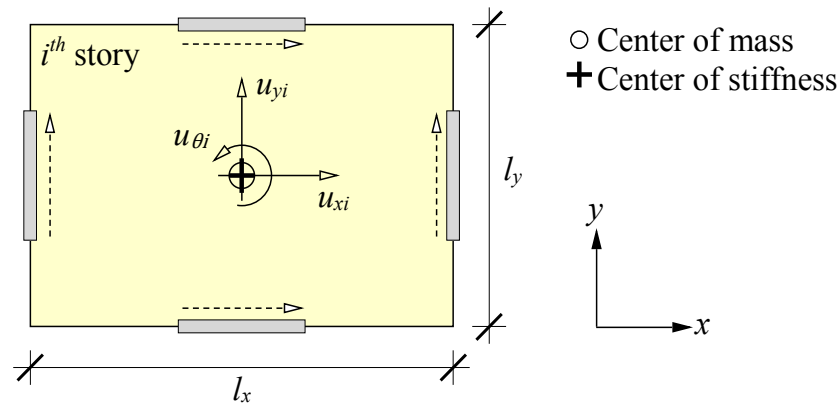


Fig. 2 – Schematic plan view of structural system with degrees of freedom denoted; thick lines indicate walls.

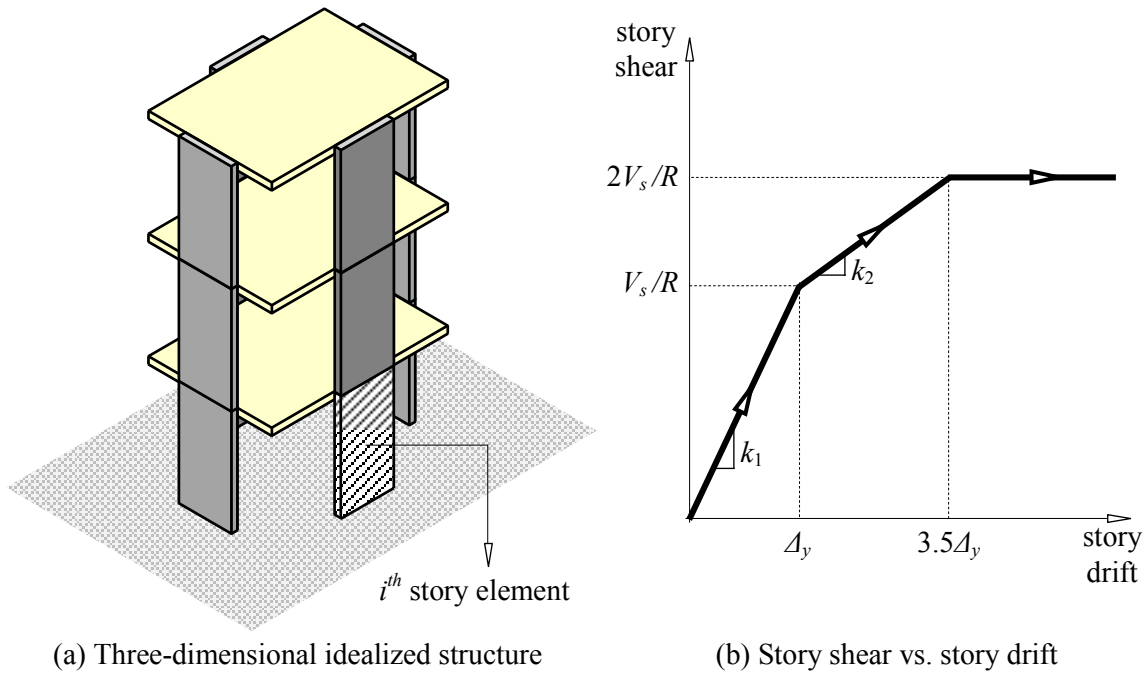


Fig. 3 – (a) Schematic isometric view of an idealized reinforced concrete structure, (b) trilinear constitutive model for vertical elements.

Table 2 – Structure parameters assumed in the analyses.

Parameter	Description	Values
$T_n$	Fundamental vibration period (s)	0.54, 1.00, 1.45, 1.87
$R$	Response modification coefficient	Linear, 3, 5, 7

#### 4. Methodology

Optimizing nonlinear RHA by modifying the input records involves three steps: (1) trimming leading weak signal, (2) trimming trailing weak signal, and (3) downsampling the trimmed record. Although the trimming of leading weak signal may change the initial conditions of the remaining acceleration time series, such changes are almost negligible; therefore they are ignored. The parameter to identify leading and trailing segments of the signal to be trimmed is the maximum roof displacement of an equivalent SDF system. This parameter is selected over other candidates such as Arias intensity [13], because it represents the characteristics of both the ground motion and structural response. The steps of the procedure are explained below.

##### Step – 1: Trimming leading weak signal

The leading weak signal that includes the pre-event interval starts from the beginning of the record to the last zero crossing before the roof displacement ( $u_r$ ) reaches an initial target roof displacement ( $u_{ri}$ ) defined as:

$$u_{ri} = f_i \times \max(|u_r(t)|) \quad (1)$$

where  $f_i$  is the displacement modification factor for the leading signal, and  $| |$  is the absolute value operator.  $u_r$  is computed by implementing UMRHA [9]]. Identification of the leading weak signal is illustrated in Fig. 4, where the top panel shows ground-motion acceleration record and the bottom panel displays  $u_r$ .

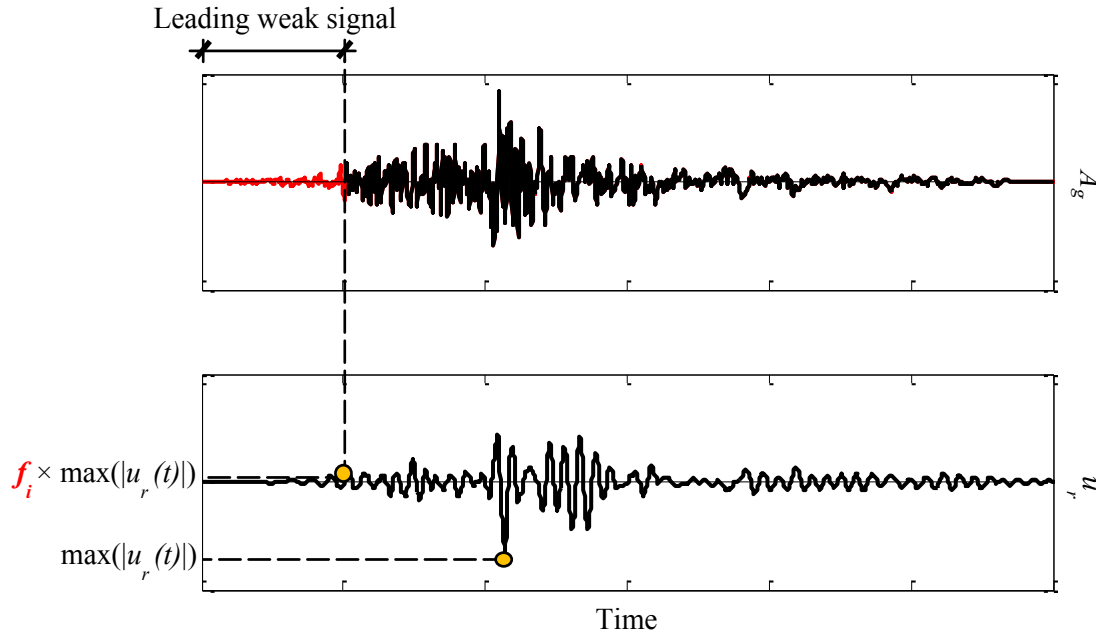


Fig. 4 – Trimming leading weak signal from the acceleration record (top panel) using modified peak roof displacement, estimated from equivalent SDF system, as a proxy (bottom panel). Maximum roof displacement and its modified value by  $f_i$  are marked with yellow circles.

### Step – 2: Trimming trailing weak signal

The trailing weak signal starts from a time instant when the roof displacement ( $u_r$ ) reaches a final target roof displacement ( $u_{rf}$ ) and ends at the termination of the record.  $u_{rf}$  is defined as:

$$u_{ri} = f_f \times \max(|u_r(t)|) \quad (1)$$

where  $f_f$  is the displacement modification factor for trailing signal. Identification of the trailing weak signal is illustrated in Fig. 5.

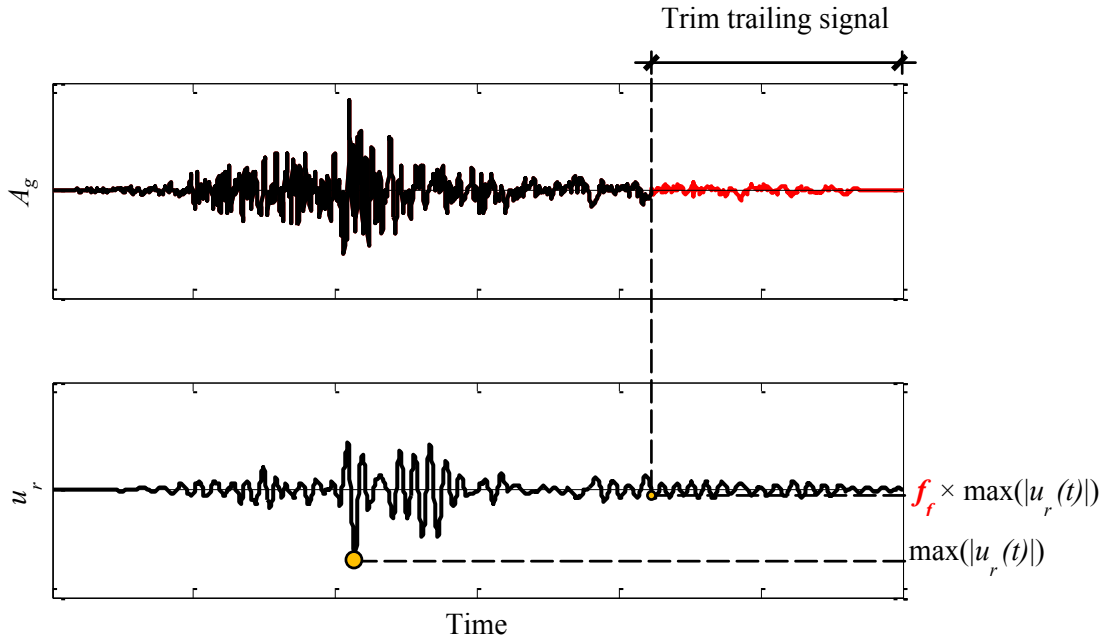


Fig. 5 – Trimming trailing weak signal from the acceleration record (top panel) using modified peak roof displacement, estimated from equivalent SDF system, as a proxy (bottom panel). Maximum roof displacement and its modified value by  $f_f$  are marked with yellow circles.

### Step – 3: Downsampling

The downsampling is performed following [7] as:

1. Transform the roof displacement time series from time domain to frequency domain;
2. Identify the largest frequency ( $\omega_{1\%}$ ) associated with an amplitude at least of 1% of the peak response;
3. Apply a low-pass filter to the trimmed record with cutoff frequency,  $\omega_{cut} = \omega_{1\%} \div f_m$ , where  $f_m$  is a modification factor that modifies the usable frequency range.
4. Modify time step  $d_t$  as a multiple of 0.005 s less than or equal to  $\pi/\omega_{cut}$  to eliminate aliasing.
5. Resample the filtered record by picking every  $m^{th}$  sample, where  $m$  represents the new sampling rate.

An example is illustrated in Fig. 6 showing the correspondence between the original and downsampled acceleration waveforms.

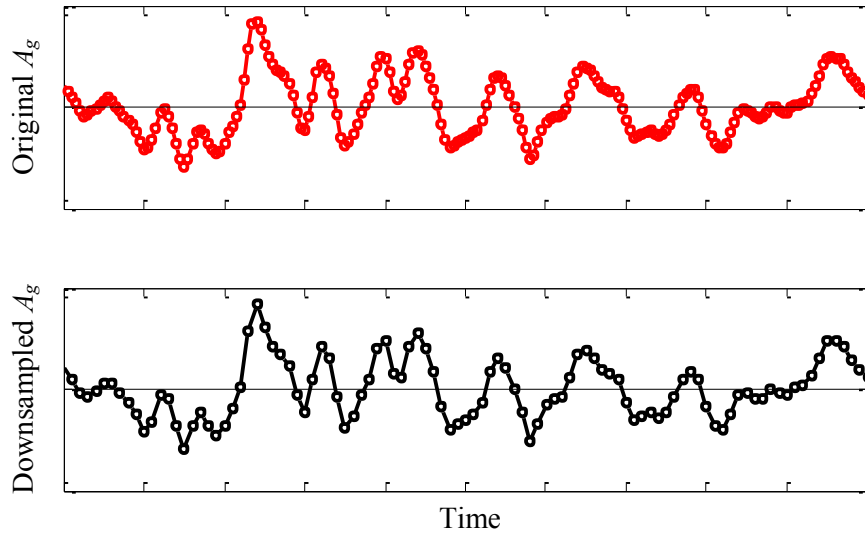


Fig. 6 – Reducing sampling rate of the acceleration record.

## 6. Results

Trimming the beginning and end segments and downsampling the remaining record may result in discrepancies in the estimates of structural response. In order to optimize the processing time while keeping the discrepancies in acceptable limits, a parametric study is conducted using seven modified records for sixteen different structural systems with various design strength ( $R$  values) and fundamental periods. The parameters ( $f_i$ ,  $f_f$  and  $f_m$ ) were varied from 0% to 50% by an interval of 2.5%.

Fig. 7 presents the relative error in peak roof displacement by trimming the leading weak signal in input records. Each column of the figure shows the results for one of the defined fundamental vibration periods, and all the plots contain series representing the obtained outcome for the selected response modification coefficients. As expected, the error in peak roof displacement is larger as the value of  $f_i$  increases because it leads to a longer segment of the record to be cut. There is no clear correlation between the error in peak displacement and the response modification coefficients. The error increases for longer periods compared to those for shorter ones. For example, if  $f_i$  is equal to 30%, the error can reach a value of 4% for structures with fundamental period of 1.00 s while it can be as high as 20% for structures with period equal to 1.87 s. In general, the error is less than 5% for values of  $f_i$  from 0% to 15%. It can be also seen that the time steps saved is almost independent of  $R$ . The results show an almost linear increment of the time step saved for small values of  $f_i$  and a constant behavior for greater values of  $f_i$ .

Fig. 8 presents the relative error in peak roof displacement by trimming the trailing weak signal in input records. The error in the estimation of the peak roof displacement is zero because the trimming of weak trailing signal occurs after the time instant when the peak value of roof displacement is reached. The time step saved is slightly affected by  $R$ , and it reduces when the fundamental period of the structure increases. Based on these results, an appropriate value of  $f_f$  could be between 20% and 25%, because it produces time step savings of 20%-40%.

The reduction of the sampling rate of the trimmed record is the most effective step of the procedure. The definition of the new time step implies that the number of points that define the record will be divided by an integer greater than or equal to one. The optimum time step occurs for  $f_m = 10\%$ , which results in 50% reduction in processing time as shown in Fig. 9. The results for other EDPs and additional structures are available in [14].



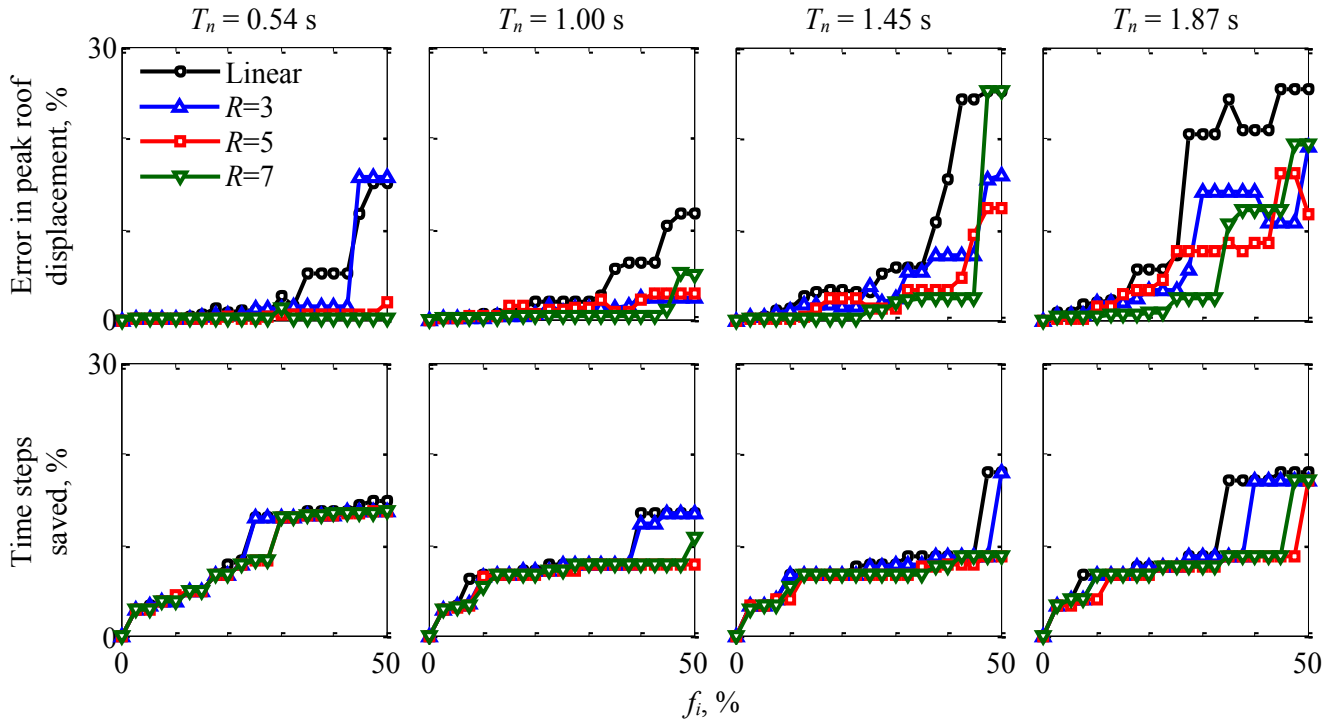


Fig. 7 – Error in median peak roof displacement (top panel), and time-step savings (bottom panel) as a function of displacement coefficient  $f_i$  used for trimming leading weak signal considering four different fundamental periods ( $T_n$ ) and  $R$  values; results are based on seven selected records shown in Fig. 1.

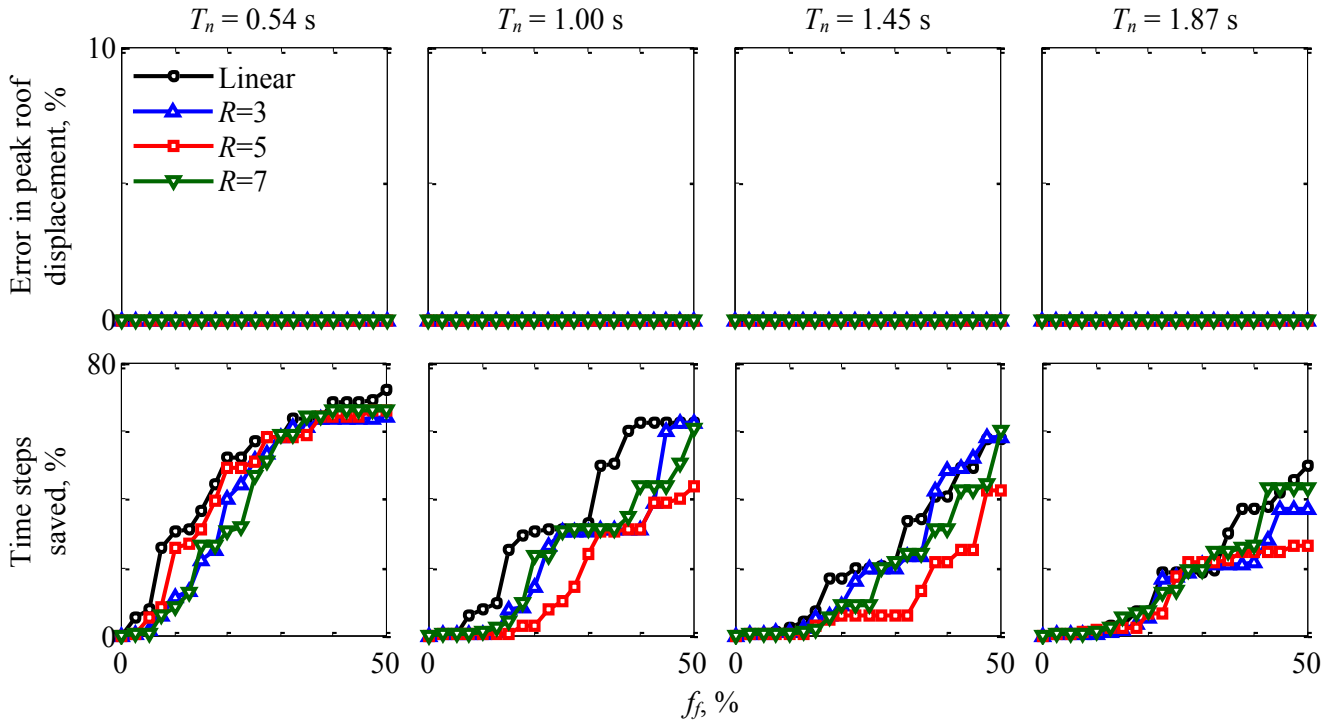


Fig. 8 – Error in median peak roof displacement (top panel), and time-step savings (bottom panel) as a function of displacement coefficient  $f_f$  used for trimming trailing weak signal considering four different fundamental periods ( $T_n$ ) and  $R$  values; results are based on seven selected records shown in Fig. 1.



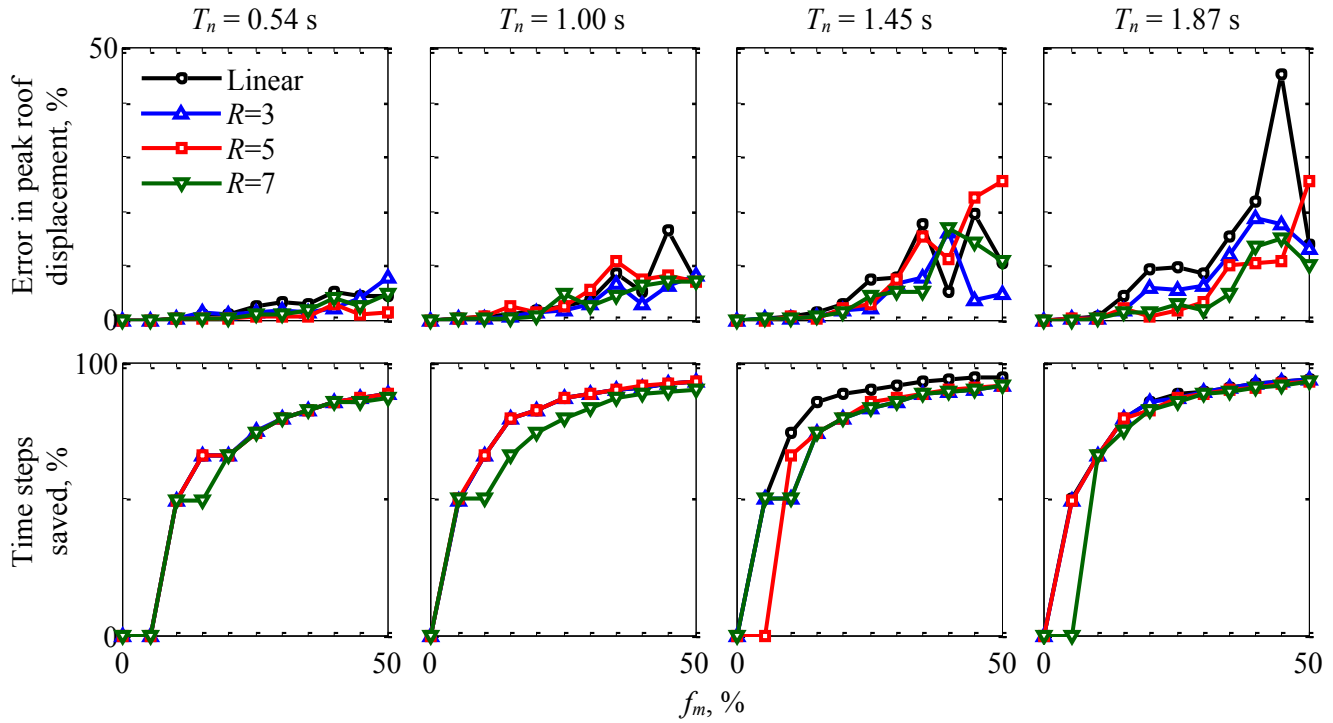


Fig. 9 – Error in median peak roof displacement (top panel), and time-step savings (bottom panel) as a function of  $f_m$  (reducing sampling rate) considering four different fundamental periods ( $T_n$ ) and  $R$  values; results are based on seven selected records shown in Fig. 1.

The goal of the above investigation is to identify an appropriate and efficient trimmed segment length for trailing and leading weak signals and optimum time step, given a ground motion and structural system, which should guarantee that there is enough data to execute an accurate nonlinear RHAs with an acceptably small and stable error. Fig. 10 demonstrates the combined effects of trimming leading and trailing weak signals and downsampling the modified records on estimates of the peak roof displacement. For optimum values of  $f_i = 10\%$ ,  $f_f = 20\%$  and  $f_m = 10\%$ , the error is within 5% and the average time steps saved is 60%.

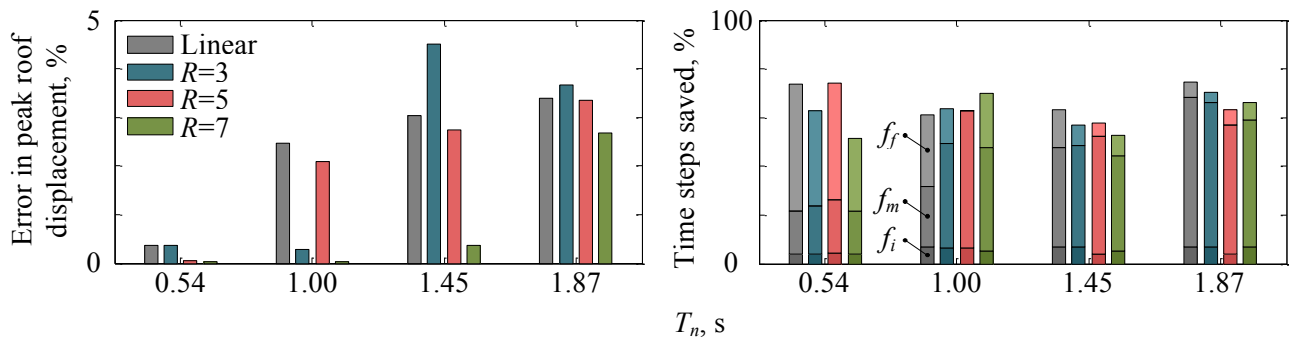


Fig. 10 – Error in peak roof displacement (in percent) and time-step savings (in percent) considering four different fundamental periods ( $T_n$ ) and response modification factors ( $R$ ) for the optimum values of  $f_i, f_f$  and  $f_m$ .

## 7. Conclusions

We propose a practical method to achieve fast nonlinear response history analysis (RHA) of multi-degrees-of-freedom (MDF) system. In this method, ground motion records are represented by a relatively short duration and reduced sampling rate. The records are trimmed from the beginning and end by removing leading and trailing weak signals. This process ensures that the *S*-phase is preserved in the trimmed record. Within a limited parametric space, we also illustrate how to estimate an optimum time step.

The method is shown to be successful in limiting the error in peak roof displacement estimates in most cases. The goodness of the approximation was measured in terms of the ability to represent the response of MDF systems, from linear to nonlinear, subjected to seven ground-motion records. The simple method introduced here needs a further validation. Additional testing results from various EDPs such as floor accelerations, floor velocities, story-drift and story-shear will be provided, and the method also extended to bi-directional excitations, in [14].

## 8. Acknowledgements

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## 9. Data and Resources

The MatLAB function of the ground-motion modification procedure and an example are available at [www.uniandes.edu.co/~jureyes/RCT.zip](http://www.uniandes.edu.co/~jureyes/RCT.zip) (last accessed Oct. 2016). The ground motions used in this study can be downloaded from <http://ngawest2.berkeley.edu/> (last accessed Oct. 2016).

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