# Development of Z number-based Fuzzy Inference System to Predict Bearing Capacity of Circular Foundations

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**Abstract**

Precise bearing capacity prediction of circular foundations is essential in civil engineering design and construction. The bearing capacity is affected by factors such as depth, density of soil, internal angle of friction, cohesion of soil, and foundation radius. In this paper, an innovative perspective on a fuzzy inference system (FIS) was proposed to predict bearing capacity. The uncertainty of fuzzy rules is eliminated by using Z-number theory. The effective parameters i.e., depth, density of soil, internal angle of friction, cohesion of soil, and foundation radius were considered as inputs to the proposed model. To compare regression and FIS model with Z-based FIS, statistical indices such as the coefficient of determination ($R^2$), root mean square error (RMSE), and variance account for (VAF) were employed. For training and testing Z-FIS, the $R^2$ was (0.977 and 0.971), the RMSE was (1.645 and 1.745), and the VAF was (98.549% and 98.138), whereas for the FIS method, the values were (0.912 and 0.904), (5.962 and 6.76), and (90.12% and 88.49%). It should be mentioned that Z theory decreased the computational time by 89.28% (174.04 s to 18.65 s). The comparison of the statistical indicators of the presented models revealed the superiority of the Z-FIS model over the FIS. Notably, sensitivity analysis revealed that the most effective parameters on bearing capacity are internal angle of friction, depth, and soil density.

**Keywords:** Bearing capacity; Circular foundation; Z-number theory; Fuzzy inference system; Prediction.

## 1. Introduction

In the case of static load on foundation, the bearing capacity has been widely studied by soil mechanic researches over the past years. The original lessons have been begun by Prandtl, (1920) and Terzaghi (Terzaghi, n.d.); subsequently, Meyerhof, (1951 and 1974), Brinch Hansen, (1961 and 1970) and Vesic, (1973) have calculated the static bearing capacity, considering the results of water surface, geometry, slope, depth, eccentricity, and load inclination. In parallel, the bearing capacity for seismic condition has often been consaudered by other methods, such as an equivalent pseudostatic method and reduction coefficients (Tiznado A & Paillao, 2014). The equivalent pseudostatic technique was used to express the bearing capacity factors by the dynamic internal friction angle (Puri & Prakash, 2007). Additionally, Meyerhof, (1951) and Shinohara et al., (1963) used a pseudostatic attitude based on acceleration in different directions such as vertical and horizontal, as gravity applied on the structure's center, so that this problem modificated to the static case with eccentric inclined load (Soubra, 1999). In dynamic load, this seismic capacity has not yet been studied for cohesive soil, even after Northridge earthquakes in 1994, Kocaeli in 1999, and Chi-Chi in 1999 (Bray & Sancio, (2006), Martin et al., (2004)). Some aspects were not taken into account based on structural failure on cohesive soil during earthquake, and little research has been done until now. In the earlier investigations, only the dynamic bearing capacity has been studied in granular soils under liquefaction. In this situation, Marcuson, (1978) found this phenomenon as the transformation of granular soil from a solid mode to a liquefied approach based on the increased pore water pressure, which reduces effective stress absolutely.

One of the most popular foundations is the ring type because of the reduced material, which has been generally exposed in several structures such as water storage tanks, silos, bridge piers, transmission towers, chimneys, and TV antennas. In terms of the bearing capacity for the ring footing, it seems to be that the limited investigations have been carried out in this way. Small scale modelling on sand soils has been tested to conclude the bearing capacity for ring footing (Boushehrian & Hataf, 2003; Saha, 1978). Likewise, the stress characteristic method (SCM) has been well completed to calculate the bearing capacity factor $N\gamma$ for smooth and rough ring foundations as interacted by sandy soils (Kumar & Ghosh, 2005), but the stress at the inner and outer edges of the ring has not been simulated. The variation of the friction angle along the interface of the footing and underlying soil mass has been implied by an approximate performance. On the basis of FLAC and by assuming an associative flow rule, the bearing capacity factor $N\gamma$ for smooth and rough ring foundations on sand has been investigated by

Zhao & Wang, (2008). For both flue rules, such as associative and non-associative, the FLAC program has also been applied to obtain $N\gamma$ when the ring foundation is based on a smooth or rough type (Benmebarek et al., 2012). The lower and upper bounds of the finite element limit analysis have been carried out to consider the bearing capacity factors, such as Nc, Nq, and N, on a ring foundation (Kumar & Chakraborty, 2015). Lately, for undrained conditions, the bearing capacity factor Nc has been investigated using the FLAC program (Remadna et al., 2017), as well as by the finite element code PLAXIS program (Lee, Jeong, & Lee, 2016; Lee, Jeong, & Shang, 2016). As noted earlier, the SCM has often been implemented to compute quite accurate solutions for different geotechnical stability problems (Bakhtavar et al., 2020). Several studies have been well done to figure out how well ring foundations will be able to hold up. This has been done through using the plastic stress field approach constructed by some methods, such as the method of characteristics (Kumar & Ghosh, 2005), limit equilibrium theory (Karaulov, 2005, 2006), finite difference method (Benmebarek et al., 2012; Zhao & Wang, 2008), and finite element method (Choobbasti et al., 2010; Lee, Jeong, & Shang, 2016). The ring plate on sands model has also been used in some tests in the laboratory (Ohri et al., 1997). Besides, some efforts have been completed to analyze the geotechnical stability of ring foundations on reinforced soil. El Sawwaf and Nazir (2012) also looked at how well the ring foundation could hold up under loads that were not straight.

For slope situations, the ultimate bearing capacity of a foundation has been performed using various techniques, such as the limit equilibrium method (Castelli & Motta, 2010; Mizuno et al., 1960), limit analysis method (Chakraborty & Kumar, 2013; Choobbasti et al., 2010), and the stress characteristic method (Graham et al., 1988). The upper-bound limit analysis process avoids the elastic-plastic body deformation and directly solves the load and velocity distribution regarding the limit state, which simplifies the challenging problem. Hence, it has become the most extensively employed method for researchers to study the ultimate bearing capacity of the foundation. In this research, the calculation of the ultimate bearing capacity is mostly based on the academic method. In the case of soil complexity, the uncertainty of the boundary assumption and the restriction of the calculation means that use of the theoretical technique to solve the problem or achieve the calculation accuracy is often problematic.

Researchers in the past often used the simplified foundation for homogeneous soil when figuring out the ultimate bearing capacity. This can make the ultimate bearing capacity smaller than it really is. Current research results in the non-homogeneity of the clay soil as an important influence on the bearing capacity of a foundation on level ground (Gourvenec & Randolph, 2003; Wai & CHEN, 1975). Many techniques, including the method of characteristics (Davis & Booker, 1973), upper-bound limit analysis method (Al-Shamrani, 2005; Reddy & Srinivasan, 1970), and numerical analysis method (Lee, Jeong, & Shang, 2016) are applied to analyze the impact of non-homogeneity on the bearing capacity. Researchers have recently used optimisation methods (Algin, 2016; Momeni et al., 2014) to figure out how much weight a foundation can hold. These methods have worked well.

In this paper, a Fuzzy inference system (FIS) is developed to predict bearing capacity. The main novely of this study is use Z-number reliability for overcoming uncertainty in the expert view in the determining fuzyy rules. The proposed approach was first introduced that is capable of sucssefulness increase accuracy level of models and decrease computaional times. This perspective of FIS can be updated for other expert-based models.

## 2. Methodology

### *2.1. Fuzzy Set*

Zadeh (L. A. Zadeh, 1975) firstly proposed the fuzzy set as a mathematical theory to confront uncertainty and vagueness in real-life world problems. The advantage of fuzzy set theory in the over of uncertainty and ambiguity of human cognitive processes is evident, and from this perspective, it differs from the classical notion. Assume $X$ be the universe of discourse, X = {$x_1$, $x_2$, . . ., $x_n$}, a fuzzy set $\tilde{a}$ of $X$ is determined by a membership function $\mu_{\tilde{a}}(x)$, which maps each component $x$ in $X$ to an actual number within the interval [0,1]. The function value $\mu_{\tilde{a}}(x)$ indicates the degree of membership of $x$ in a. The higher $\mu_{\tilde{a}}(x)$, the bigger the degree of membership for $x$ in $\tilde{a}$ .There have been a lot of concerns raised about fuzzy sets in various uses; for example, failure mode and effects analysis (Bakhtavar et al., 2021), fuzzy fault tree analysis (Jiskani et al., 2022), mine blasting (Bakhtavar et al., 2017), industry 4.0 (Poormirzaee et al., 2022b, 2022a), occupational hazards in underground mines (Hosseini et al., 2022), risk analysis (Bakhtavar et al., 2020), green mining (Bakhtavar et al., 2019). Fuzzy sets are introduced briefly in the following definitions.

#### 2. 1. 1. Fuzzy numbers

A fuzzy number demonstrates a unique fuzzy set in the universe of discourse X, which membership function related to it is both normal and convex. Fuzzy logic employs various types of fuzzy membership functions including trapezoidal fuzzy numbers (TFNs) and triangular fuzzy numbers (TrFNs) (as shown in Figure 1). Nevertheless, triangular fuzzy numbers are applied to reveal experts' opinions in this research because they are more effective in applications and more practical in improving reproduction and knowledge processing in a fuzzy environment. Let $\tilde{a}$ be a TFNs, $\tilde{a}$ =($a_1$, $a_2$, $a_3$), where membership function $\mu_{\tilde{a}}(x)$ can be determined as:

$$\mu_{\tilde{a}}(x)=\begin{cases} 0 \quad , & 0 < a_1 \\ \dfrac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ \dfrac{a_3-x}{a_3-a_2}, & a_2 \le x \le a_3 \\ 0 \quad , & a_3 > 0 \end{cases} \tag{1}$$

where $a_1$, $a_3$, and $a_2$ are the lower bound, upper bound, and the modal value of the fuzzy number $\tilde{a}$ , respectively.

Similarly, the membership of a TrFNs, $\tilde{a}$ , can be defined by a quadruplet ($a_1$, $a_2$, $a_3$, $a_4$) as follows:

$$\mu_{\tilde{a}}(x)=\begin{cases} 0 \quad , & 0 < a_1 \\ \dfrac{1}{a_2-a_1}x-\dfrac{a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ 1 \quad , & a_2 \le x \le a_3 \\ \dfrac{-1}{a_4-a_3}x+\dfrac{a_4}{a_4-a_3}, & a_3 \le x \le a_4 \\ 0 \quad , & a_4 > 0 \end{cases} \tag{2}$$

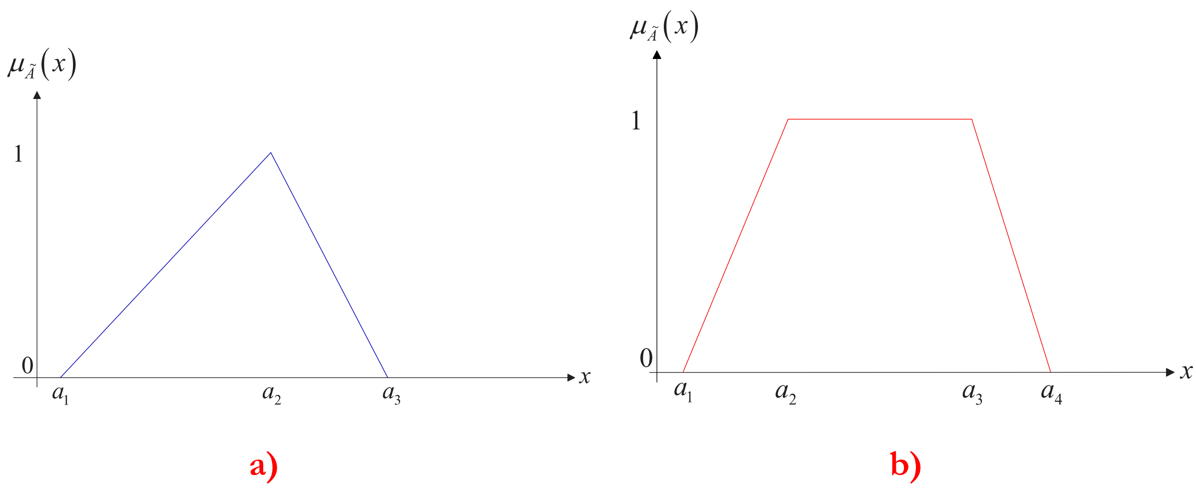



Figure 1. Fuzzy number: a) TFN, b) TrFN

Assume $\tilde{a}$ = ($a_1$, $a_2$, $a_3$), $\tilde{b}$ = ($b_1$, $b_2$, $b_3$) are two positive TFNs and $r$ is a positive real number; the arithmetic operations of the TFNs can be performed by:

| | | |
|---|---|---|
| Addition: | $a \oplus b = \left(a_1 + b_1,\, a_2 + b_2,\, a_3 + b_3\right)$ | (3) |
| Subtraction: | $a \ominus b = \left(a_1 + b_1,\, a_2 + b_2,\, a_3 + b_3\right)$ | (4) |
| Multiplication: | $a \otimes b \cong \left(a_1 b_1,\, a_2 b_2,\, a_3 b_3\right)$ | (5) |
| Multiplication of any real number r and a TFN: | $r \otimes b \cong \left(a_1 b_1,\, a_2 b_2,\, a_3 b_3\right)$ | (6) |
| Division: | $a \oslash b \cong \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right)$ | (7) |

**2. 1. 2. Linguistic variables**

A linguistic variable refers to a variable whose values are words or sentences in a natural or artificial language, which is very applicable in trading with too complicated or too ill-defined conditions to be wisely expressed by conventional quantitative opinions. These variables can also be described in the form of fuzzy numbers. Common linguistic terms with their fuzzy numbers and crisp value tabulated in Table 1. Also, Figure 2 illustrates their membership functions for visualization.

Table 1. A scale for linguistic variables and TFNs (Bakhtavar et al. 2019)

| Crisp value | Linguistic variable | | TFNs |
|---|---|---|---|
| 1 | Very low | VL | $\tilde{1} = (1,1,3)$ |
| 3 | Low | L | $\tilde{3} = (1,3,5)$ |
| 5 | Medium | M | $\tilde{5} = (3,5,7)$ |
| 7 | High | H | $\tilde{7} = (5,7,9)$ |
| 9 | Very high | VH | $\tilde{9} = (7,9,9)$ |

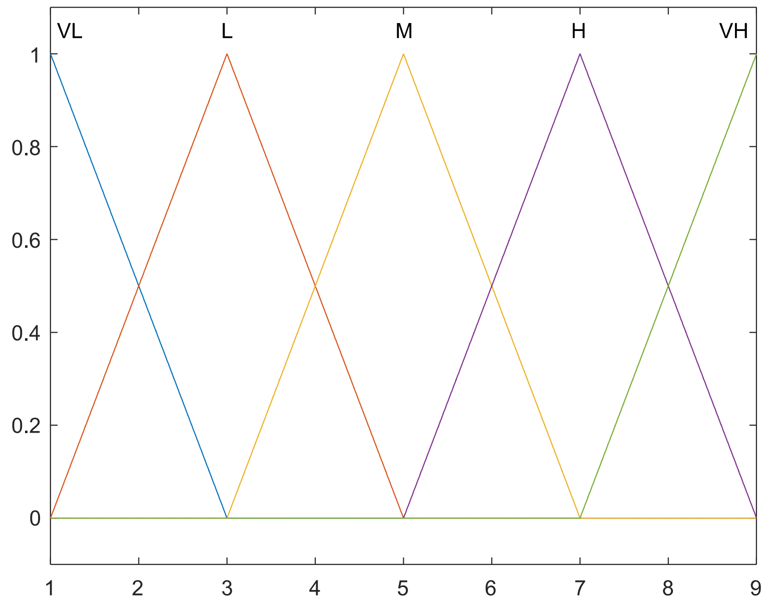



Figure 2. Membership function of criteria

**2. 1. 2. Defuzzification**

Defuzzification is a necessary action in the fuzzy process to determine the best nonfuzzy performance (BNP) value. Several ways for this aim are presented, such as the mean of maxima (MOM), center of area (COA), and α-cut. Various defuzzification techniques extract various levels of information. COA method is a practical and straightforward way that does not need to bring any preferences of decision-makers. Hence, this method is implemented in this study to find out the BNP value. Let $\tilde{a}$=($a_1$, $a_2$, $a_3$) and $\bar{x}_0(\tilde{a})$ be fuzzy number and defuzzified value of the fuzzy number $\tilde{a}$, respectively. The BNP value of the TFN can be calculated by:

$$\bar{x}_0(\tilde{a}) = \frac{1}{3}\left\{\left(a_3 - a_1\right) + \left(a_2 - a_1\right)\right\} + a_1 \tag{8}$$

***2.2. Z-numer Concept***

The initial idea of Z-numbers for modeling uncertain information was first proposed by Zadeh in 2011 (Zadeh, 2011). Mahler (Mahler, 1968) also proposed the notion of Z-numbers in 1968, which is different from Z-numbers introduced by Zadeh. Z-numbers are ordered pairs of fuzzy numbers [$Z = (\tilde{A}, \tilde{B})$], which is defined as an uncertain variable Z-values by Zadeh. $\tilde{A}$ as the first component of Z indicates restriction on a real variable X. Nevertheless, $\tilde{B}$ as the second component of Z denotes the reliability of the first component. Zadeh (Zadeh, 2011) defined Z-numbers as follows inherent meaning:

$$Z = (A, B) = Z^{+}\left(A, \mu_A \cdot p_{X_A} \text{ is } B\right) \tag{9}$$

A simple Z-number is shown as Figure 3.

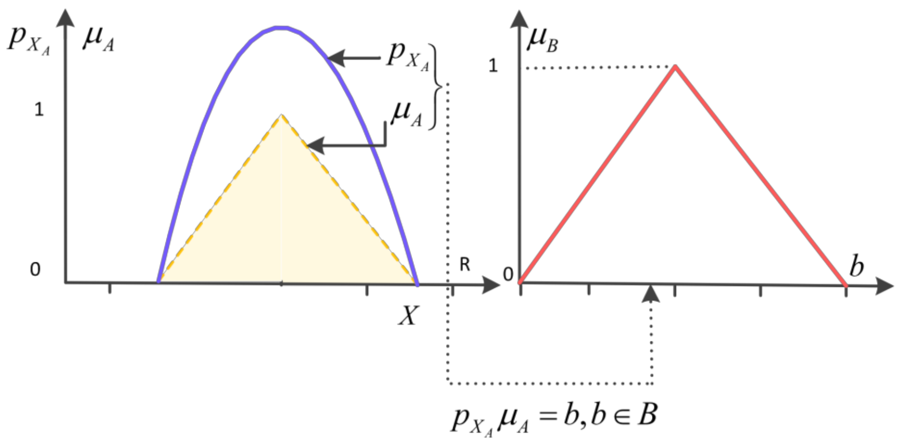



Figure 3. A simple Z-number

Based on literature reviews, Z-numbers based theories are more reliable compared to uncertainty problems. Z-number theory can be illustrated by the following example:

The bearing capacity in construction project is reported as follows:

"bearing capacity rate in a depth of 12 meter is about 4355 kg/cm$^2$", very high. This report can be defined as "X is Z = (A, R)." In contrast, X is the "bearing capacity rate in a depth of 12 meter " expression, A is a fuzzy set that announcements the bearing capacity rate "12 meter", and R is the reliability level of A if it is "very high" (Equation 5). The probability restriction can be showed by Equation (10):

$$R(X) = X \quad \text{is } A \tag{10}$$

In which, A indicates probability distribution X. The probability restriction can be better announcement as:

$$R(X): X \text{ is } A \rightarrow Poss\ (X = u) = \mu_A(u) \tag{11}$$

In which, u and μA stand the real values of X and membership function of A, respectively. In this regard, a restriction can be showed for set of A as R(X):

$$R(X): X \text{ is } p \tag{12}$$

Where p denotes probability density function of X. Therefore, Equation (11) is rewritten as:

$$R(X): X \text{ is } p \rightarrow Prob\ (u \le X \le u + du) = p(u)du \tag{13}$$

#### 2. 1. 2. Linguistic Z-Number Operations

The operations of Z-numbers are too complex; therefore, Wang et al. (Wang et al., 2017) presented possible arithmetics operations for Z-numbers. The proposed operations consider both the flexibility of linguistic variable sets and the reliabilities measure of Z-values. Let $Z_1 = (A_1, R_1)$ and $Z_2 = (A_2, R_2)$ be two linguistic Z-numbers. The functions of $f^*$ and $g^*$ can be considered from between $f_1(s_l)$, $f_2(s_l)$, $f_3(s_l)$ and $f_4(s_l)$. Therefore, several operations in the linguistic Z-numbers environment can be presented as follows:

$$neg(z_1) = \left(f^{*-1}\left(f^*(A_{2m}) - f^*(A_1)\right), g^{*-1}\left(g^*(R_{2m}) - g^*(R_1)\right)\right) \tag{14}$$

$$z_1 + z_2 = \left(f^{*-1}\left(f^*(A_1) + f^*(A_2)\right), g^{*-1}\left(\frac{f^*(A_1)\times g^*(R_1) + f^*(A_2)\times g^*(R_2)}{f^*(A_1) + f^*(A_2)}\right)\right) \tag{15}$$

$$\lambda z_1 = \left(f^{*-1}\left(\lambda f^*(A_1)\right), R_1\right) \quad , \quad \lambda \ge 0 \tag{16}$$

$$z_1 \times z_2 = \left(f^{*-1}\left(f^*(A_1) f^*(A_2)\right), g^{*-1}\left(g^*(R_1) g^*(R_2)\right)\right) \tag{17}$$

$$z_1^{\lambda} = \left(f^{*-1}\left(f^*(A_1)^{\lambda}\right), g^{*-1}\left(g^*(R_1)^{\lambda}\right)\right) \quad , \quad \lambda \ge 0 \tag{18}$$

Furthermore, assume $Z_1 = (A_1, R_1)$ be a linguistic Z-number. The accuracy function and score function of linguistic Z-numbers is determined as Equations (19) and (20):

$$A(z_1) = f^*(A_1)\times\left(1 - g^*(R_1)\right) \tag{19}$$

$$S(z_1) = f^*(A_1)\times g^*(R_1) \tag{20}$$

Suppose that $Z_1 = (A_1, R_1)$, $Z_1 = (A_2, R_2)$ and $Z_3 = (A_3, R_3)$ be three linguistic Z-numbers, and $f^*$ and $g^*$ be linguistic fuzzy sets. Then, the following properties are true:

$$z_1 \oplus z_2 = z_2 \oplus z_1; \tag{21}$$

$$z_1 \otimes z_2 = z_2 \otimes z_1; \tag{22}$$

$$\lambda\left(z_1 \oplus z_2\right) = \lambda z_1 \oplus \lambda z_2 \ , \ \lambda > 0; \tag{23}$$

$$\left(z_1 \otimes z_2\right)^{\lambda} = z_1^{\lambda} \otimes z_2^{\lambda}; \tag{24}$$

$$\lambda_1 z_1 \oplus \lambda_2 z_1 = \left(\lambda_1 + \lambda_2\right) z_1 \ , \ \lambda_1 \geq 0, \lambda_2 \geq 0; \tag{25}$$

$$z_1^{\lambda_1} \otimes z_1^{\lambda_2} = z_1^{\left(\lambda_1 + \lambda_2\right)} \ , \ \lambda_1 \geq 0, \lambda_2 \geq 0; \tag{26}$$

$$\left(z_1 \oplus z_2\right) \oplus z_3 = z_1 \oplus \left(z_2 \oplus z_3\right) \tag{27}$$

$$\left(z_1 \otimes z_2\right) \otimes z_3 = z_1 \otimes \left(z_2 \otimes z_3\right) \tag{28}$$

**2. 1. 2. Converting Z-numbers to crisp numbers**

As a more description, we are indicate how Z-number sets translated into regular fuzzy numbers. Assume $Z = (\tilde{A}, \tilde{R})$ as a Z-number and let triangular membership functions be as follows:

$$\left\{\tilde{A} = \left(x, \mu_{\tilde{A}}\right) \mid x \in [0,1]\right\} \tag{29}$$

$$\left\{\tilde{B} = \left(x, \mu_{\tilde{B}}\right) \mid x \in [0,1]\right\} \tag{30}$$

Conserning to reliability level of first component of Z-number, second component (reliabilities) is transformed into a crisp number by using Equation (31):

$$\alpha = \frac{\int x \mu_{\tilde{B}}(x)\ dx}{\int \mu_{\tilde{B}}(x)\ dx} \tag{31}$$

Then, crisp value of reliabilities are considered in the restriction part of Z-number:

$$\tilde{Z}^{\alpha} = \left\{ \left( x, \mu_{\tilde{A}} \right) | \mu_{\tilde{A}^{\alpha}}\left( x \right) = \alpha \mu_{\tilde{A}}\left( x \right), x \in [0,1] \right\} \tag{32}$$

Finally, Z-numberS (weighted restriction) converted into the fuzzy numbere $\tilde{Z}'$:

$$\tilde{Z}' = \left\{ \left( x, \mu_{\tilde{A}'} \right) | \mu_{\tilde{A}'}\left( x \right) = \mu_{\tilde{A}}\left( \frac{x}{\sqrt{\alpha}} \right), x \in [0,1] \right\} \tag{33}$$

If $\tilde{A} = (L, M_1, M_2, U)$ is a TrFNs, then $\tilde{Z}'$ is determined as:

$$\tilde{Z}' = \left( \sqrt{\alpha} \cdot L, \sqrt{\alpha} \cdot M_1, \sqrt{\alpha} \cdot M_2, \sqrt{\alpha} \cdot U \right) \tag{34}$$

We will show conversions of Z-number, $Z = (\tilde{A}, \tilde{R})$, using a numerical example; if the opinion of an expert (A) and his reliability (R) be follows:

$\tilde{A} = (0.5, 0.6, 0.7, 0.8; 1)$

$\tilde{R} = (0.4, 0.5, 0.6; 1)$

The opinion of Expert can be illustrate to Z-number as

$$\tilde{Z} = \left( \tilde{A}, \tilde{R} \right) = \left[ \left( 0.5, 0.6, 0.7, 0.8 \right), \left( 0.4, 0.5, 0.6 \right) \right] \tag{35}$$

Firstly, the reliability part is transformed to a crisp value as follows:

$$\alpha = \frac{\int x \mu_{\tilde{B}}(x)\ dx}{\int \mu_{\tilde{B}}(x)\ dx} = 0.5$$

Secondly, the constraint is weighted by reliability (α) as follows:

$\tilde{Z}^{\alpha}$ =(0.5,0.6,0.7,0.8; 0.5)

Third, transform the weighted Z-number to regular fuzzy number:

$$\begin{aligned} \tilde{Z}' &= \left( \sqrt{0.5} \times 0.5, \sqrt{0.5} \times 0.6, \sqrt{0.5} \times 0.7, \sqrt{0.5} \times 0.8; 1 \right) \\ &= \left( 0.707 \times 0.5, 0.707 \times 0.6, 0.707 \times 0.7, 0.707 \times 0.8; 1 \right) \\ &= \left( 0.354, 0.424, 0.495, 0.566; 1 \right) \end{aligned}$$

The weighted Z-number after converting to regular fuzzy number is illustared in Figure 4.

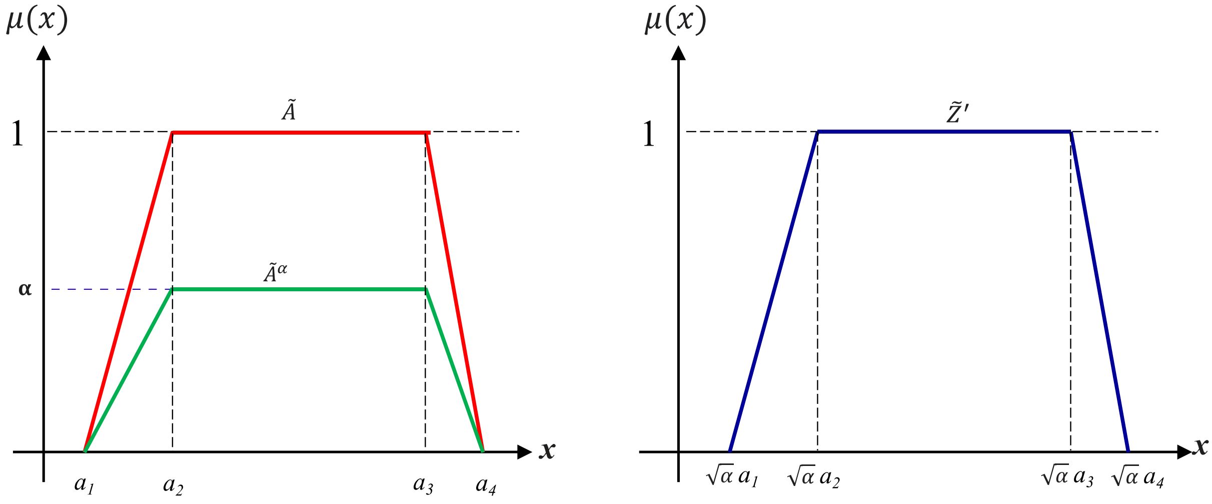



Figure 4. Transforming of Z-number to regular fuzzy number.

### *2.3. Fuzzy Inference System*

As aforementioned, fuzzy set theory was first introduced by (Zadeh, 1965). This theory satisfied approximately a mathematical solution to solve complicated judgment problems with intuitive, imperfect, and inaccurate knowledge, which classical methods are not able efficiently to explain them. This theory can process all types of information varying from interval-valued numerical data to linguistics terms (Dubois & Parde, 2000). The obtain of fuzzy models from observed or estimated information has recently gained increasing attention. Fuzzy sets theory considers a unsertationy of human decisions and reflect the overview of real world; therefore, this sets has more applications compared to the classic sets (Shams et al., 2015). The fuzzification is a process to define membership functions of related to fuzzy variables, which knowlege of experts is used for determination of membership function. Then, all inputs are transformed into degree of memberships according to relevant appropriate membership function (Yagiz & Gokceoglu, 2010). In the fuzzy theory, different types of membership functions such as sigmoidal (psigmf), gaussian (gaussmf), gaussian combination (gauss2mf), triangular (trimf), trapezoidal (trapmf), linear s-shaped saturation (linsmf), linear z-shaped saturation (linzmf), Pi-shaped (pimf), S-shaped (smf), Z-shaped (zmf), difference between two sigmoidal (dsigmf), and product of two sigmoidal (psigmf) employes to express ligustic terms (see Figure 5). Fuzzy sets use membership functions to represent mathematically linguistic terms of uncertainty such as “extremely low (EL)“, “very low (VL)“, “Low (L)“, “medium low

(ML)“, “medium (M)“, “medium high (MH)“, “high (H)“, “very high (VH)“, and “extremely high (EH)“.

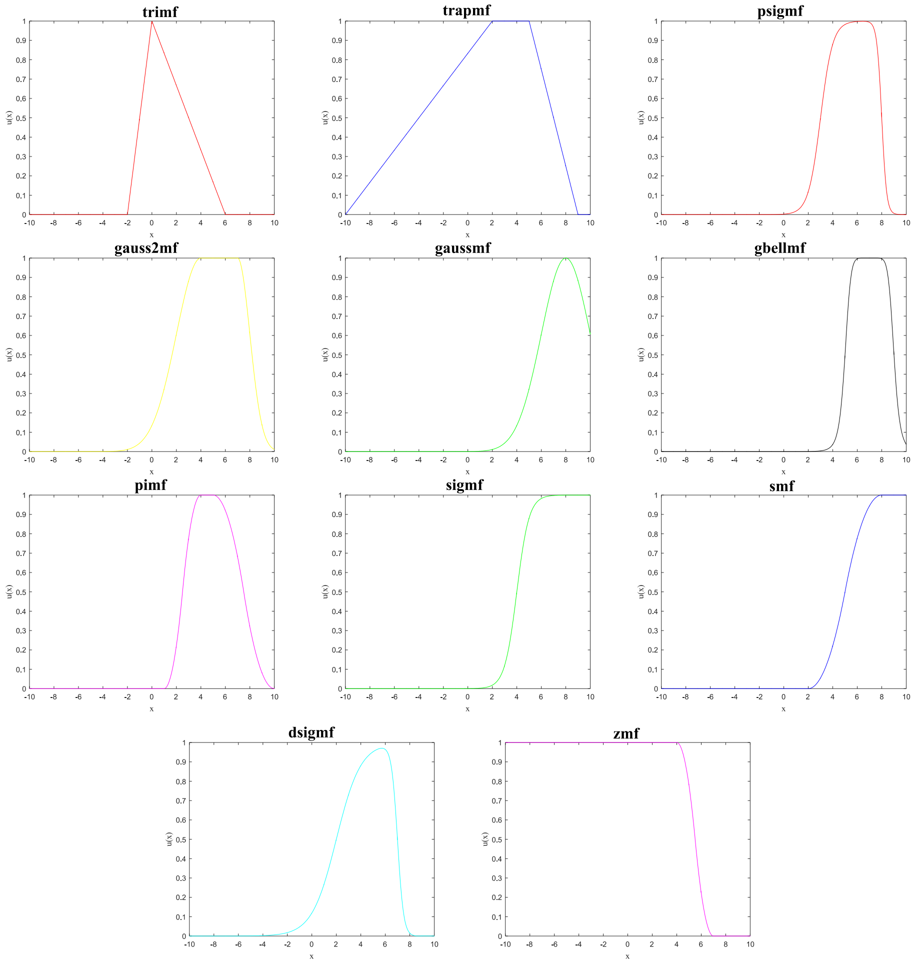



Figure 5. Various type of membership functions

FIS is an applicable computational tool capable of decision and classification examinations (Galetakis and Vasiliou 2010), which consists of three main layers: fuzzification layer, reasoning engine layer, and defuzzification layer (Figure 6). In the first step, the crisp inputs are imported into the fuzzifier system, and fuzzy inputs are generated. In this regard, knowledge bases are employed to system forward. In the second step, different rules are defined, and a rule base is constructed to use in the system. Then, a database is employed to determine membership functions. In the third step, fuzzy information is processed in the inference engine based on a reasoning mechanism, and finally, logic or crisp output is obtained. In fact, fuzzy rules revealed the relations between input(s) and output(s) data, which structured the FIS model for describing complicated and imprecise systems. This fuzzy process is performed to construct a rule-based model, in which fuzzy if-then rules (or implication functions) are used instead of fuzzy propositions. Therefore, the principle portion of a FIS model is a rule-based model restructured by combining experts' knowledge and numerical information.

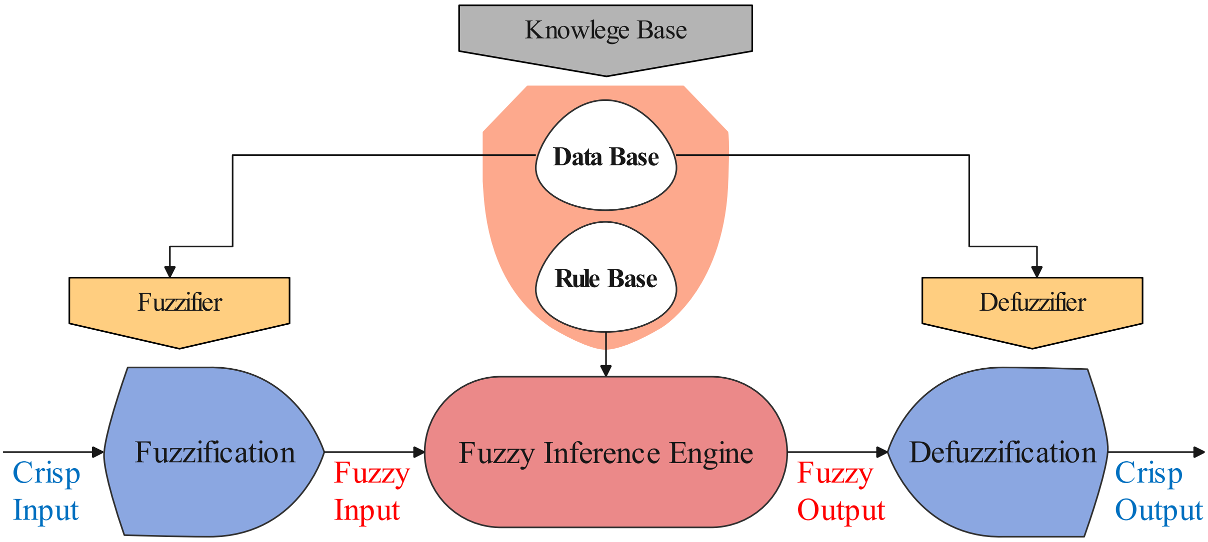



Figure 6. Fuzzy inference engine

An element's membership is always crisp when it is part of a classic or ordinary set; thereby, there are two types of elements: those that belong to a set and those that don't. It suffices to represent each member of these sets with an only unique membership functions. Whereas, a sharp boundaries do not defined for the fuzzy sets as a generelized ordinary sets; hence, the degree of an element in a set can range in the interval [0, 1] (see Figure 7).

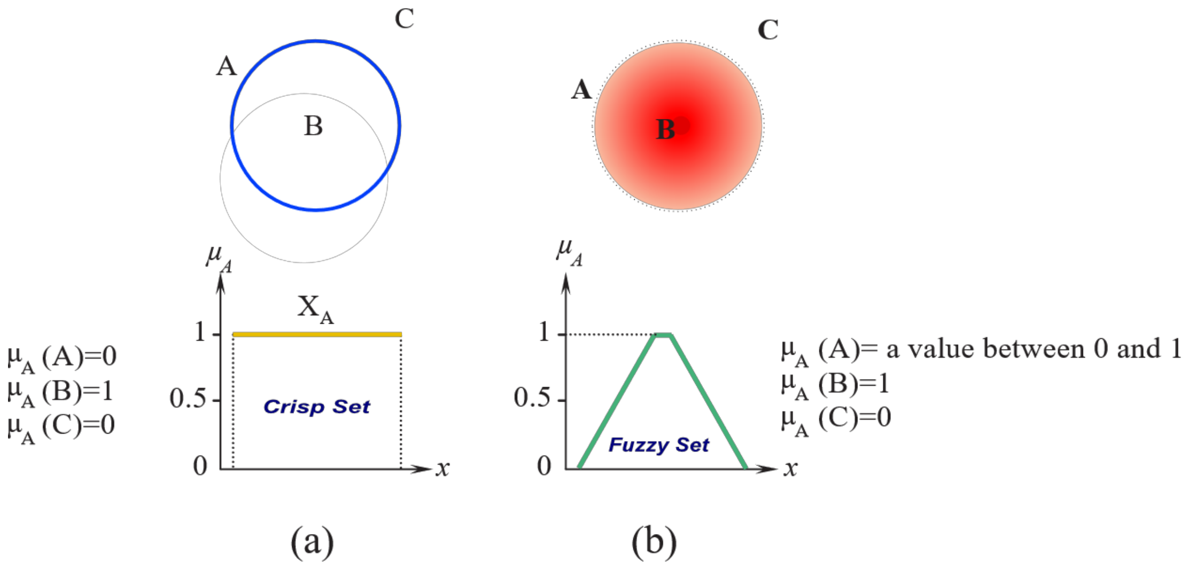



Figure 7. a) Crisp set and b) fuzzy set

The process of combining the individual consequents into a single fuzzy set or final consequent is called aggregation of rules. Aggregation is the process by which the fuzzy sets (individual consequents) are combined into a single fuzzy set (final consequent) is named aggregation. Aggregation occurs before the final defuzzification step by using a maximum operator that relevant input and output are the truncated output functions and fuzzy sets, respectively. The aggregation process is widely performed by applying following models (Iphar & Goktan, 2006):

- Mamdani fuzzy model,
- Takagi–Sugeno–Kang fuzzy (TSK) model,
- Tsukamoto fuzzy model, and
- Singleton fuzzy model.

In the fuzzy logic, Mamdani fuzzy model is one oft he most applicable and known algorithm among four abovementioned models (Iphar & Goktan, 2006). Based on this model, totally unstructured set of linguistic heuristics can be transformed into structured algorithms by using fuzzy sets and fuzzy logic (Mamdani & Assilian, 1975). Mamdani "if-then" rule structure is generally formed as follows:

If $x_i$ is $A_{il}$…and $x_r$ is $A_{ir}$ then $y$ is $B_i$ (i=1, 2, …, $k$)

where $x_i$ stands input parameter, $y$ denotes output parameter and $k$ indicates the number of rules (Sonmez et al., 2003).

Among different composition methods of Mamdani FIS, Min-Max operation is the most widely used technique. A Mamdani fuzzy model with two relus is illustrated in Figure 8, in which, "z" represents overall system output and "x" and "y" are crisp inputs. For each rule, the consequential fuzzy set is trimmed through the minimum of the prototype fuzzy sets utilizing the minimum operator.

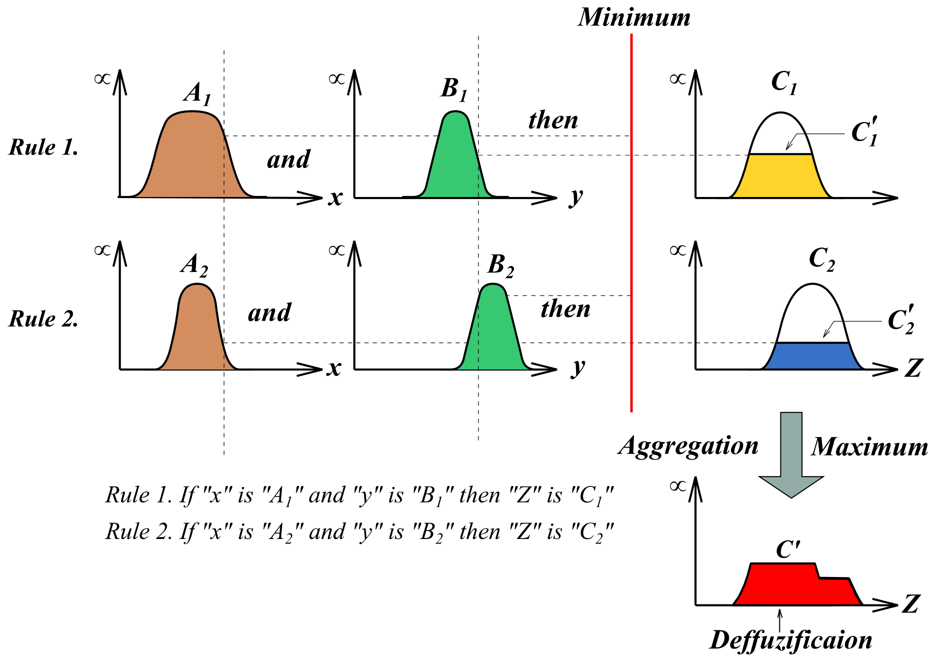



Figure 8. The Mamdani FIS using Min-Max composition method

In the FIS model, final step is defuzzification of fuzzy outputs; fuzzy sets is converted into crisp values. Noteworthy, there exists the various defuzzification methods including:

- Centroid
- Bisector
- Middle of maximum
- Smallest of maximum
- Largest of maximum

The centroid of area (COA) is the most frequently employed technique among other methods in the FIS (Grima, 2000). The COA method calculated the crisp value as below:

$$Z^*_{COA} = \frac{\int_z \mu_A(z) z \, dz}{\int_z \mu_A(z) dz}$$

where $Z^*_{COA}$ specify the crisp values of output ("$z$"), and $\mu_A(z)$ is the aggregated output membership function.

## 3. Laboratory Tests and Database Preparation

The datasets achieved in the laboratory were used to develop the models in this study. The device for conducting direct shear test is shown in Figure 9. This device is used to identify soil resistance parameters such as cohesion and internal angle of friction. The database used for developing models in this study is tabulated in Table 1. The descriptive statistics of effective parameters and bearing capacity for 968 data are summarized in this table. Figure 10 represents correlations between the parameters used for the development models. As can be found from Figure 10, the correlation between D, DS, IAF and the BC are approximately good with the carrelations of 0.57, 0.648, and 0.709, respectively, while the correlation between the FR and CS with the BC are very low with the carrelations of -0.807, and 0.281, respectively. Furthermore, the correlation between the FR and other parameters are very weak. The correlation between CS with DS and IAF, D with IAF, and CS with BC were negantive.

Figure 9. Direct shear test

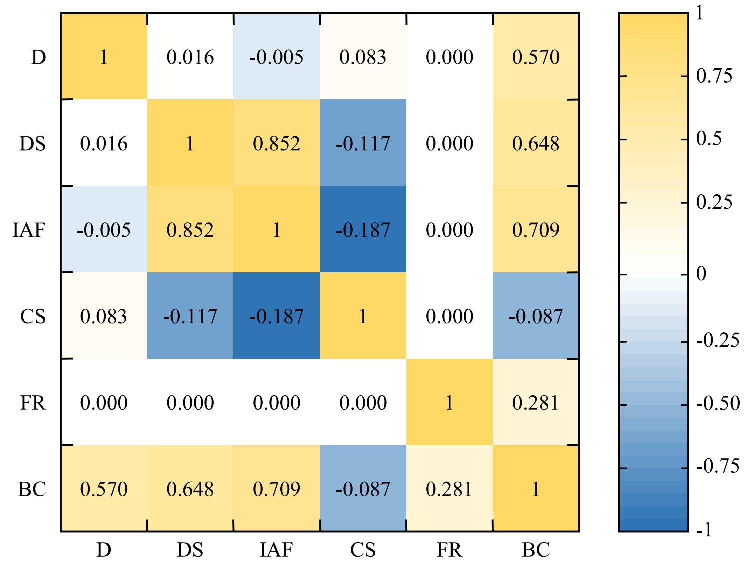



Figure 10. The correlations between inputs and output

The frequency histogram of the BC is presented in Figure 11. As can be seen, 303 data are accompanied by a BC in the interval (645.06,1056.10] kg/cm2; nevertheless, a BC in the interval (3933.39,4344.43] kg/cm2 is observed in 3 BC data.

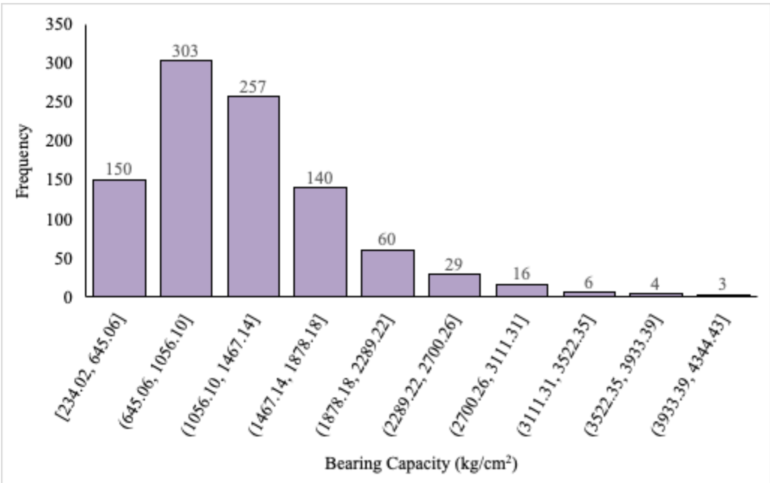



Figure 11. Frequency histogram of the BC

Table 1. Descriptove statistics of parameters

| Parameters | Depth | Density of soil | Internal angle of friction |
|---|---|---|---|

| | | | |
|---|---|---|---|
| Symbol | D | DS | IAF |
| Unit | m | $gr/cm^3$ | degree |
| | Min:2.5, Mean:7.34, Max:13, Std:3.25 | Min:1.57, Mean:1.76, Max:2.04, Std:0.12 | Min:21.1, Mean:25.83, Max:34.5, Std:2.85 |
| Parameters | Cohesion of Soil | Foundation Radius | Bearing Capacity |
| Symbol | CS | FR | BC |
| Unit | $kg/cm^2$ | m | $kg/cm^2$ |
| | Min:0.03, Mean:0.11, Max:0.2, Std:0.05 | Min:10, Mean:27.88, Max:45, Std:11.83 | Min:234.02, Mean:1214.19, Max:4344.43, Std:613.24 |

# 4. Results and Analysis

## *4.1. Statistical model for bearing capacity*

The multivariate regression (MR) method was used to construct a statistical model. In this regard, the relationships between effective parameters and output parameter as respectively independent and dependent parameters are established. In this study, BC is determined by using product of the five independent parameters, i.e., D, DS, IAF, CS, and FR. The SPSS V. 25 is employed to obtain a regression predictive model for the forecast of BC (Eq (36)). The statistical information concerning the constituted predictive model is presented in Table 2.

$$\begin{aligned} BC = -4449.441 + \left(107.843 \times D\right) + \left(618.219 \times \ DS\right) \\ + \left(131.078 \times IAF\right) + \left(CS \times \left(-85.430\right)\right) + \left(14.579 \times FR\right) \end{aligned} \tag{36}$$

where D is Depth (m), DS is Density of soil ($gr/cm^3$), IAF is Internal angle of friction (degree), CS is Cohesion of Soil ($kg/cm^2$), FR is Foundation Radius (m), and BC is Bearing Capacity ($kg/cm^2$)

Table 2. MR results for predition of BC

| Independentvariable | Coefficients | Standard Error | t Stat | p value |
|---|---|---|---|---|
| Intercept | -4449.441 | 97.372 | -45.695 | 0.000 |
| D | 107.843 | 1.773 | 60.831 | 0.000 |
| DS | 618.219 | 93.322 | 6.625 | 0.000 |
| IAF | 131.078 | 3.908 | 33.541 | 0.000 |
| CS | -85.429 | 124.153 | -0.688 | 0.042 |
| FR | 14.580 | 0.485 | 30.030 | 0.000 |

R squared = 1 − (residual sum of squares) / (corrected sum of squares) = 0.725

### *4.2. Fuzzy model for bearing capacity*

As beforementioned, the Mamdani structure was used to establish fuzzy model and develop BC predictive model. The parameters of depth, density of soil, internal angle of friction, cohesion of soil, and foundation radius were imported as inputs of the fuzzy model to estimate bearing capacity as model output.

As beforementioned, the Mamdani structure was used to establish fuzzy model and develop BC predictive model. The parameters of depth, density of soil, internal angle of friction, cohesion of soil, and foundation radius were imported as inputs of the fuzzy model to estimate bearing capacity as model output. The fuzzy structure with the imported input and output parameters in the model is shown in Figure 12. In the first step of FIS modeling, the input parameters is fuzzified using most fit membership functions. For this aim, gaussian (gaussmf) and gaussian combination (gauss2mf) membership functions as the most usable membership functions were applied to fuzzification parameters.

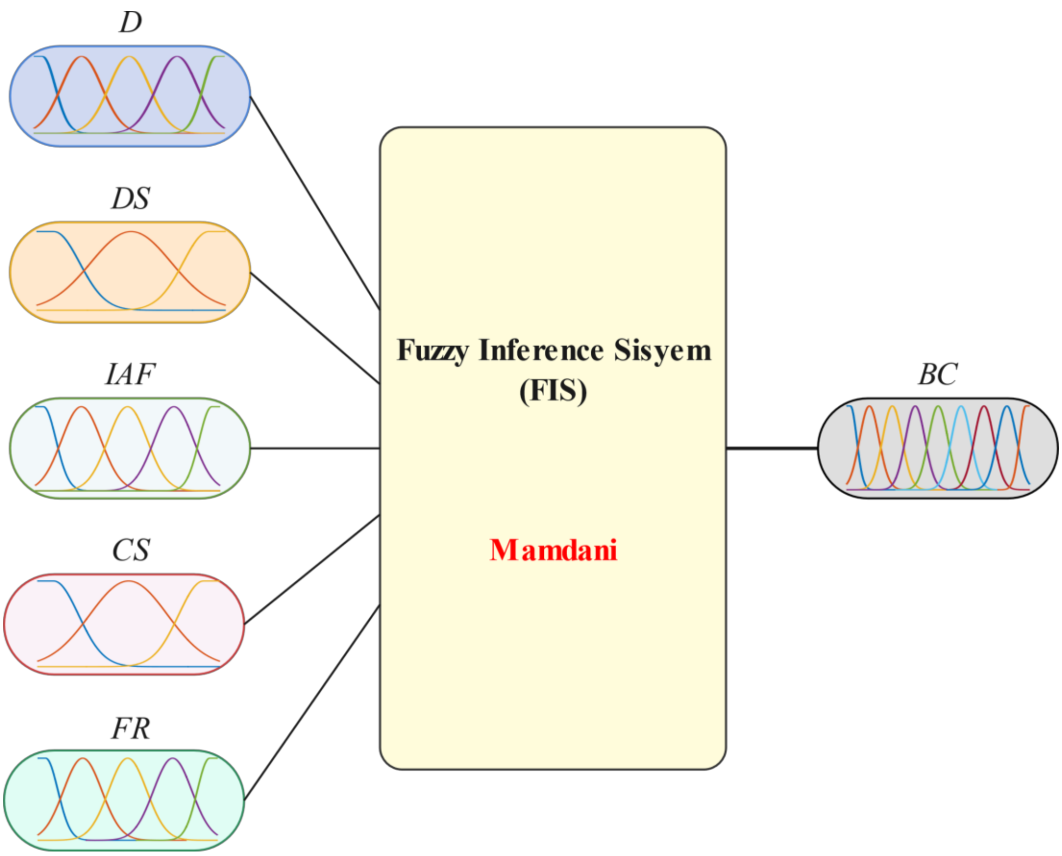



Figure 12. Schematic illustration of the fuzzy inference model

Here, the linguistic terms with nine categories were defined as “extremely low (EL)“, “very low (VL)“, “Low (L)“, “medium low (ML)“, “medium (M)“, “medium high (MH)“, “high

(H)“, “very high (VH)“, and “extremely high (EH)“. Notably, the degrees of membership for parameters are selected according to experts' knowledge and experiences. In addition, the number of membership functions was widely obtained based on the trial and error procedure. The “underfitting” (requisite accuracy occurs) and “overfitting” (mendacious accuracy occurs) problems are the consequences that are respectively accrued due to the insufficient and excessive number of rules.

Based on abovementioned expression, the membership functions of input paramaters and output parameters were specified as shown in Figure 13. In the FIS modeling, a total number of 1,125 rules were applied to developing the Mamdani-based model. It should be mentioned that this number of rules has been finalized after removing overlapped rules. Finally, the Mamdani aggregation algorithm as the widest method in FIS was used considering the problem complexity. Table 3 summarized the some of the fuzzy rules employed in the FIS modeling.

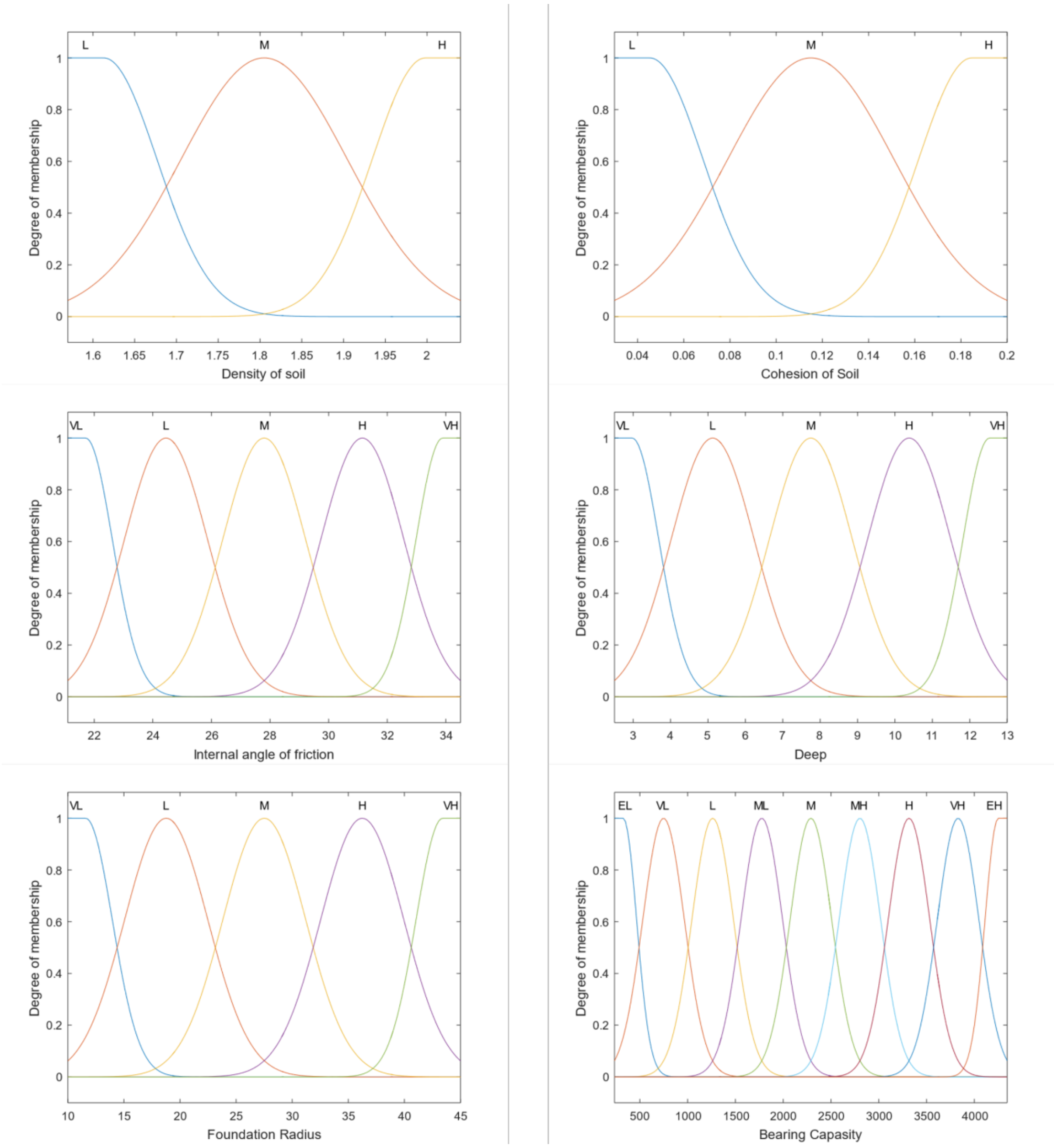



Figure 13. Membership functions of parameters

Table 3. Several examples of if-then fuzzy rules

| Rule number | Description of if-then rules |
|---|---|
| 1 | If (D is VL) and (DS of soil is L) and (IAF is VL) and (CS is L) and (FR is VL) then (BC is EL) (1) |
| 65 | If (D is VH) and (DS of soil is L) and (IAF is VH) and (CS is L) and (FR is VL) then (BC is MH) (1) |
| 91 | If (D is VL) and (DS of soil is L) and (IAF is L) and (CS is M) and (FR is VL) then (BC is EL) (1) |
| 117 | If (D is L) and (DS of soil is H) and (IAF is M) and (CS is M) and (FR is VL) then (BC is VL) (1) |
| 134 | If (D is H) and (DS of soil is H) and (IAF is H) and (CS is M) and (FR is VL) then (BC is M) (1) |
| 151 | If (D is VL) and (DS of soil is L) and (IAF is VL) and (CS is H) and (FR is VL) then (BC is EL) (1) |
| 170 | If (D is VH) and (DS of soil is L) and (IAF is L) and (CS is H) and (FR is VL) then (BC is L) (1) |
| 205 | If (D is VH) and (DS of soil is M) and (IAF is H) and (CS is H) and (FR is VL) then (BC is ML) (1) |

| | |
|---|---|
| 256 | If (D is VL) and (DS of soil is L) and (IAF is M) and (CS is L) and (FR is L) then (BC is VL) (1) |
| 349 | If (D is H) and (DS of soil is L) and (IAF is H) and (CS is M) and (FR is L) then (BC is ML) (1) |
| 453 | If (D is M) and (DS of soil is L) and (IAF is VL) and (CS is L) and (FR is M) then (BC is VL) (1) |
| 634 | If (D is H) and (DS of soil is L) and (IAF is M) and (CS is H) and (FR is M) then (BC is ML) (1) |
| 685 | If (D is VH) and (DS of soil is M) and (IAF is VL) and (CS is L) and (FR is H) then (BC is ML) (1) |
| 743 | If (D is M) and (DS of soil is M) and (IAF is VH) and (CS is L) and (FR is H) then (BC is EH) (1) |
| 786 | If (D is VL) and (DS of soil is M) and (IAF is M) and (CS is M) and (FR is H) then (BC is VL) (1) |
| 832 | If (D is L) and (DS of soil is M) and (IAF is VL) and (CS is H) and (FR is H) then (BC is VL) (1) |
| 848 | If (D is M) and (DS of soil is M) and (IAF is L) and (CS is H) and (FR is H) then (BC is L) (1) |
| 894 | If (D is H) and (DS of soil is M) and (IAF is VH) and (CS is H) and (FR is H) then (BC is H) (1) |
| 941 | If (D is VL) and (DS of soil is H) and (IAF is M) and (CS is L) and (FR is VH) then (BC is L) (1) |
| 1020 | If (D is VH) and (DS of soil is H) and (IAF is M) and (CS is M) and (FR is VH) then (BC is MH) (1) |
| 1067 | If (D is L) and (DS of soil is L) and (IAF is L) and (CS is H) and (FR is VH) then (BC is VL) (1) |
| 1110 | If (D is VH) and (DS of soil is H) and (IAF is H) and (CS is H) and (FR is VH) then (BC is H) (1) |
| 1123 | If (D is M) and (DS of soil is H) and (IAF is VH) and (CS is H) and (FR is VH) then (BC is MH) (1) |
| 1125 | If (D is VH) and (DS of soil is H) and (IAF is VH) and (CS is H) and (FR is VH) then (BC is H) (1) |

In the last step of FIS modeling, the defuzzification process is performed, in which the fuzzy values are converted into crisp values using the COA techniques. The rule viewer and fuzzy reasoning engine of the MATLAB environment are depicted in Figure 14. As can be found, when input parameters are D=7.75 m, DS=1.81 gr/cm3, IAF=22.3 degree, CS=0.0649 kg/cm2, and FR=12.7 m, then BC would be 851 kg/cm2, which is very close to measured BC with the value of 843.

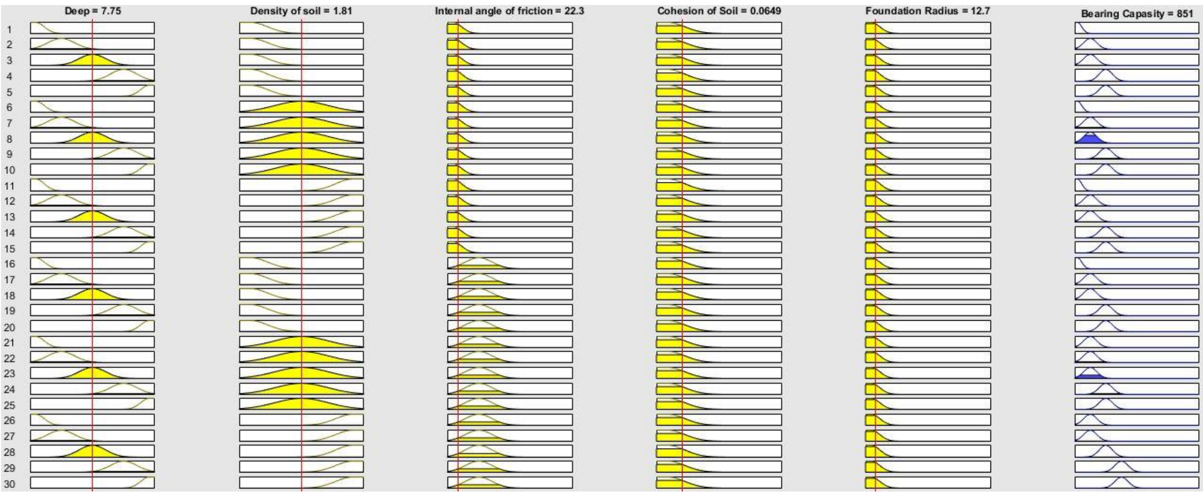



Figure 14. An example calculation for the FIS model

### *4.3. Z-number based Fuzzy model for bearing capacity*

As aforementioned, the fuzzy rules are specified based on expert knowledge. Nevertheless, expert opinions to determine fuzzy rules have uncertainty. Therefore, the membership functions identified for the output variable deal very insufficient reliability. Therefore, this study focused on implementing the Z-number concept to overcome the uncertainty of expert views. In this regard, the reliability level of Z-number is applied in the analyzing process and the range of 0-100% confidence is specified for expert use. In other words, a particular scale was defined as tabulated in Table 4 to express the judgments reliability level of experts. The membership degrees of Z-number linguistic terms are displayed in Figure 15. The confidence of 0 and 100% are applied for strong reliability and unreliability, respectively. The results can be significantly improved by this reliability level.

Table 4. The rules of transformation concerned with linguistic variables of possibilities

| Reliability | | |
|---|---|---|
| **Number** | **Linguistic terms** | **Membership function** |
| 1 | 0% sure | (0,0,0.025,0.05) |
| 2 | 5% sure | (0.025,0.05,0.075,0.1) |
| 3 | 10% sure | (0.075,0.1,0.125,0.15) |
| 4 | 15% sure | (0.125,0.15,0.175,0.2) |
| 5 | 20% sure | (0.175,0.2,0.225,0.25) |
| 6 | 25% sure | (0.225,0.25,0.275,0.3) |
| 7 | 30% sure | (0.275,0.3,0.325,0.35) |
| 8 | 35% sure | (0.325,0.35,0.375,0.4) |
| 9 | 40% sure | (0.375,0.4,0.425,0.45) |
| 10 | 45% sure | (0.425,0.45,0.475,0.5) |
| 11 | 50% sure | (0.475,0.5,0.525,0.55) |
| 12 | 55% sure | (0.525,0.55,0.575,0.6) |
| 13 | 60% sure | (0.575,0.6,0.625,0.65) |
| 14 | 65% sure | (0.625,0.65,0.675,0.7) |
| 15 | 70% sure | (0.675,0.7,0.725,0.75) |
| 16 | 75% sure | (0.725,0.75,0.775,0.8) |
| 17 | 80% sure | (0.775,0.8,0.825,0.85) |
| 18 | 85% sure | (0.825,0.85,0.875,0.9) |
| 19 | 90% sure | (0.875,0.9,0.925,0.95) |
| 20 | 95% sure | (0.925,0.95,0.975,1) |
| 21 | 100% sure | (0.975,1,1,1) |

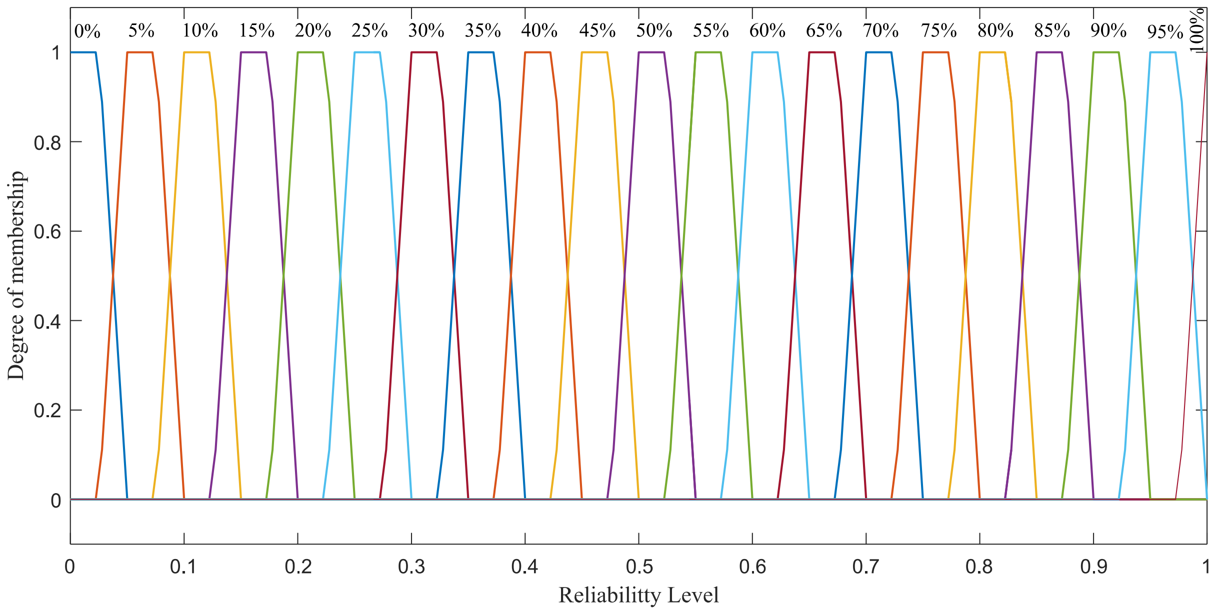



Figure 15. Membership degree of Z-numbers of transformation rules

In the first phase, the experts first determined 1125 fuzzy rules to develop the Mamdani-based FIS model. In the second phase, they also expressed their reliability level of the opinions based on 21 linguistic terms. For this aim, the scale presented in Table 5 was identified to evaluation the first component of Z- number, Z= (A, B). The Membership degree of TrFNs for A component is depicted in Figure 16. It should be mentioned that the scale shown in Table 4 is used for determining B component of Z.

Table 5. The rules of transformation concerned with linguistic variables of restrictions

| **Evaluation (A component)** | |
|---|---|
| **Linguistic term** | **Fuzzy number** |
| Extremely low (EL) | (0,0,285.4,696.4) |
| Very low (VL) | (285.4,696.4,799.2,1210) |
| Low (L) | (799.2,1210,1313,1724) |
| Medium low (ML) | (1313,1724,1827,2238) |
| Medium (M) | (1827,2238,2341,2752) |
| Medium high (MH) | (2341,2752,2854,3265) |
| High (H) | (2854,3265,3368,3779) |
| Very high (VH) | (3368,3779,3882,4293) |
| Extremely high (EH) | (3882,4293,4500,4500) |

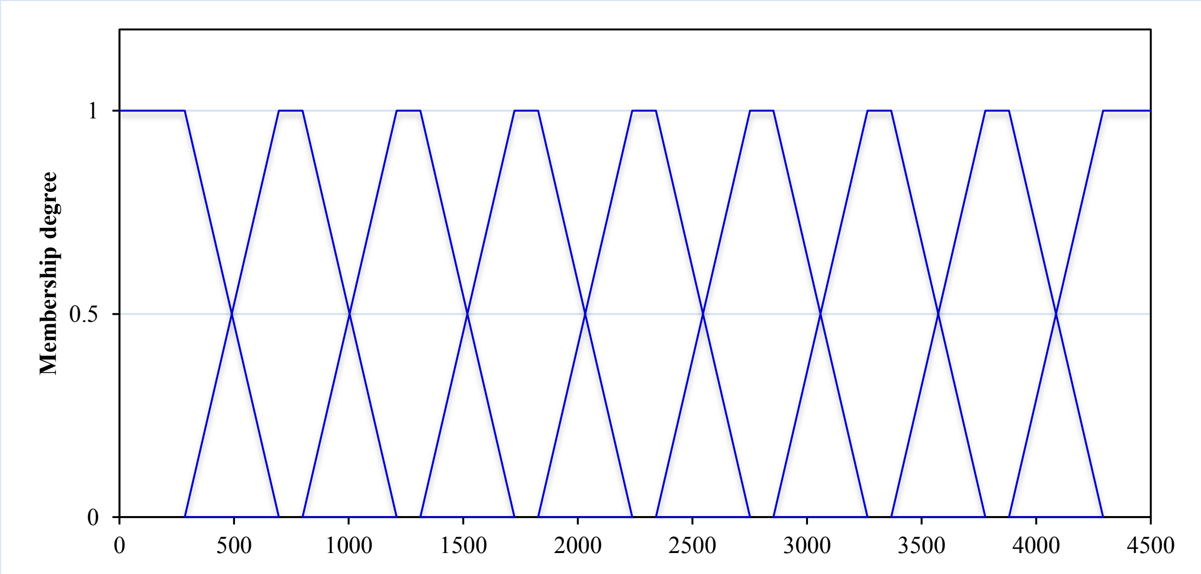



Figure 16. Membership degree of TrFNs identified for A component

Therefore, each member of Tables 4 and 5 is transformed into a regular number by repeating the procedure presented in "Converting Z-numbers to crisp numbers" section (Eq. (31)-(35)). Table 6 presents a sample of Z-calculation for the same rules summarized in Table 3. Combining translation terms (Table 5) and reliabilities related to constraints (Table 4) results in the conversion rules of linguistic variables of experts of Z-numbers. A fuzzy rating is then created based on these results. Suppose n criteria are met by the object of research for the restriction. Accordingly, the number of membership functions of output parameters is modified based on the new Z-relus. The 59 Z-based membership functions were defined for BC as shown in Figure 17. In this step, the new Z-based FIS is developed for predicting BC.

Table 6. Judgment of expert with reliability information

| Rule number | A | B | Membership function of A | Membership function of B | Linguistic phrase | Acronym | a | $\sqrt{a}$ | Z-number |
|---|---|---|---|---|---|---|---|---|---|
| 1 | EL | 90% | (0,0,0.05,0.1) | (0,0,0.15,0.25) | Extremely low - 90% sure | (EL-90%) | 0.913 | 0.955 | (0,0,272.628,665.235) |
| 65 | MH | 80% | (0,0,0.05,0.1) | (0.15,0.25,0.35,0.45) | Medium high- 80% sure | (MH-80%) | 0.813 | 0.901 | (2110.149,2480.619,2572.561,2943.031) |
| 91 | EL | 100% | (0,0,0.05,0.1) | (0.35,0.45,0.55,0.65) | Extremely low - 100% sure | (EL-100%) | 0.996 | 0.998 | (0,0,284.805,694.948) |
| 117 | VL | 90% | (0,0,0.05,0.1) | (0.55,0.65,0.75,0.85) | Very low -90% sure | (VL-90%) | 0.913 | 0.955 | (272.628,665.235,763.435,1155.851) |
| 134 | M | 90% | (0,0,0.05,0.1) | (0.75,0.85,0.95,1) | Medium-90% sure | (M-90%) | 0.913 | 0.955 | (1745.239,2137.846,2236.237,2628.844) |
| 151 | EL | 90% | (0.05,0.1,0.2,0.25) | (0,0,0.15,0.25) | Extremely low - 90% sure | (EL-90%) | 0.913 | 0.955 | (0,0,272.628,665.235) |
| 170 | L | 90% | (0.05,0.1,0.2,0.25) | (0.15,0.25,0.35,0.45) | Low -90% sure | (L-90%) | 0.913 | 0.955 | (763.435,1155.851,1254.241,1646.849) |
| 205 | ML | 80% | (0.05,0.1,0.2,0.25) | (0.35,0.45,0.55,0.65) | Medium low - 80% sure | (ML-80%) | 0.813 | 0.901 | (1183.522,1553.993,1646.836,2017.306) |
| 256 | VL | 90% | (0.05,0.1,0.2,0.25) | (0.55,0.65,0.75,0.85) | Very low -90% sure | (VL-90%) | 0.913 | 0.955 | (272.628,665.235,763.435,1155.851) |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 349 | ML | 95% | (0.05,0.1,0.2,0.25) | (0.75,0.85,0.95,1) | Medium low - 95% sure | (ML-95%) | 0.963 | 0.981 | (1288.146,1691.366,1792.416,2195.637) |
| 453 | L | 100% | (0.2,0.25,0.3,0.35) | (0,0,0.15,0.25) | Low -100% sure | (L-100%) | 0.996 | 0.998 | (797.533,1207.477,1310.262,1720.405) |
| 634 | ML | 95% | (0.2,0.25,0.3,0.35) | (0.15,0.25,0.35,0.45) | Medium low - 95% sure | (ML-95%) | 0.963 | 0.981 | (1288.146,1691.366,1792.416,2195.637) |
| 685 | ML | 90% | (0.2,0.25,0.3,0.35) | (0.35,0.45,0.55,0.65) | Medium low- 90% sure | (ML-90%) | 0.913 | 0.955 | (1254.241,1646.849,1745.239,2137.846) |
| 743 | EH | 90% | (0.2,0.25,0.3,0.35) | (0.55,0.65,0.75,0.85) | Extremely high-90% sure | (EH-90%) | 0.913 | 0.955 | (3708.275,4100.882,4298.619,4298.619) |
| 786 | VL | 90% | (0.2,0.25,0.3,0.35) | (0.75,0.85,0.95,1) | Very low -90% sure | (VL-90%) | 0.913 | 0.955 | (272.628,665.235,763.435,1155.851) |
| 832 | VL | 90% | (0.3,0.35,0.45,0.5) | (0,0,0.15,0.25) | Very low -90% sure | (VL-90%) | 0.913 | 0.955 | (272.628,665.235,763.435,1155.851) |
| 848 | L | 90% | (0.3,0.35,0.45,0.5) | (0.15,0.25,0.35,0.45) | Low -90% sure | (L-90%) | 0.913 | 0.955 | (763.435,1155.851,1254.241,1646.849) |
| 894 | MH | 90% | (0.3,0.35,0.45,0.5) | (0.35,0.45,0.55,0.65) | Medium high - 90% sure | (MH-90%) | 0.913 | 0.955 | (2236.237,2628.844,2726.28,3118.887) |
| 941 | VL | 90% | (0.3,0.35,0.45,0.5) | (0.55,0.65,0.75,0.85) | Very low -90% sure | (VL-90%) | 0.913 | 0.955 | (272.628,665.235,763.435,1155.851) |
| 1020 | MH | 65% | (0.3,0.35,0.45,0.5) | (0.75,0.85,0.95,1) | Medium high - 65% sure | (MH-65%) | 0.663 | 0.814 | (1905.436,2239.966,2322.988,2657.517) |
| 1067 | VL | 90% | (0.45,0.5,0.55,0.6) | (0,0,0.15,0.25) | Very low -90% sure | (VL-90%) | 0.913 | 0.955 | (272.628,665.235,763.435,1155.851) |
| 1110 | MH | 100% | (0.45,0.5,0.55,0.6) | (0.15,0.25,0.35,0.45) | Medium high- 100% sure | (MH-100%) | 0.996 | 0.998 | (2336.118,2746.261,2848.048,3258.191) |
| 1123 | MH | 100% | (0.45,0.5,0.55,0.6) | (0.35,0.45,0.55,0.65) | Medium high- 100% sure | (MH-100%) | 0.996 | 0.998 | (2336.118,2746.261,2848.048,3258.191) |
| 1125 | MH | 100% | (0.45,0.5,0.55,0.6) | (0.55,0.65,0.75,0.85) | Medium high- 100% sure | (MH-100%) | 0.996 | 0.998 | (2336.118,2746.261,2848.048,3258.191) |

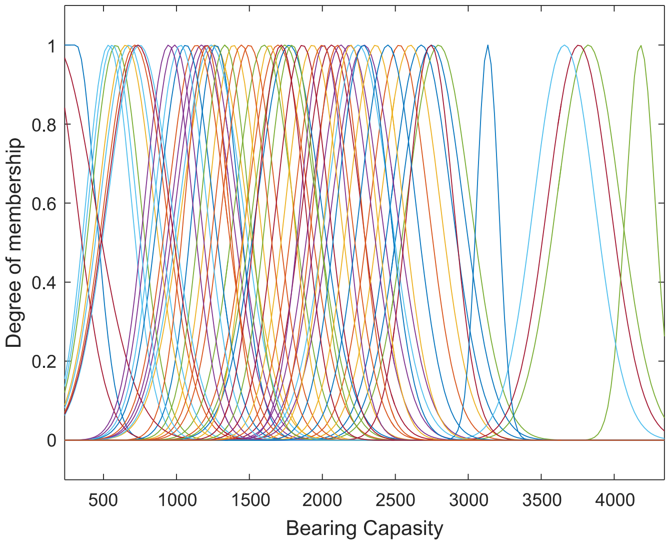



Figure 17. Z-based Membership functions of BC

### *4.3. Sensitivity analysis*

The sensitivity analysis is performed to determine the most influential input parameters on output parameter(s). In this study, the impact of each input parameter on BC was specified using the cosine amplitude method. The sensitivity is evaluated through a factor, namely '*r*' can be calculated as follows:

$$r_{ij} = \frac{\sum_{k=1}^{m}\left(x_{ik} \times x_{jk}\right)}{\sqrt{\left(\sum_{k=1}^{m} x_{ik}^2 \times \sum_{k=1}^{m} x_{jk}^2\right)}} \tag{6}$$

In which, $x_i$ stands input parameters, $x_j$ indicates output parameter(s), and n is the number of data. The impact value of each inputs on BC is illustrated in Figure 18. As can be seen, IAF, D, and DS have the most impact on BC.

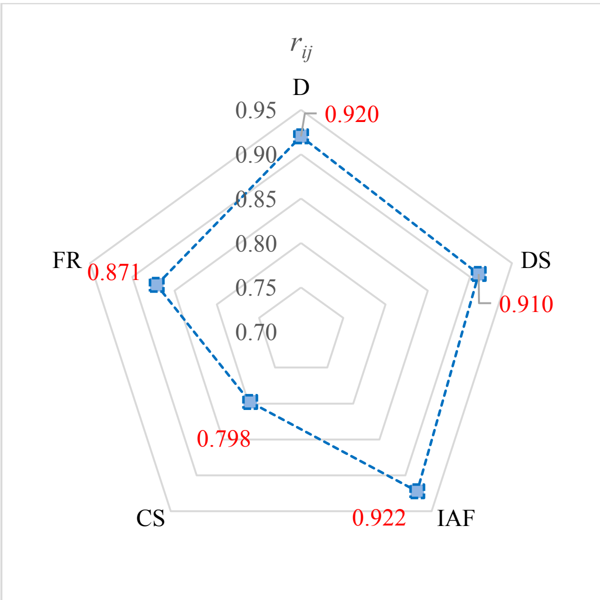



Figure 18. Sensitivity alaysis of input parameters

## 5. Results

In this study, 968 data for BC were estimated through Z-FIS, FIS, and MR Methods. Tthe dataset is first splited into two categorires: training (80% of data) and testing (20% of data). Next, the three statistical indicators--coefficient of determination ($R^2$), root-mean-squared error (RMSE), and value account for (VAF)--were calculated to compare the developed models with FIS and MR. The indicators are calculated as follows:

$$R^2 = 1 - \left( \frac{\sum_{i=1}^{n} (O_i - P_i)^2}{\sum_{i=1}^{n} (P_i - \overline{P}_i)^2} \right) \tag{37}$$

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(O_i - P_i)^2} \tag{38}$$

$$VAF = 100 \cdot \left(1 - \frac{var(O_i - P_i)}{var(O_i)}\right) \tag{39}$$

Where $O_i$ and $P_i$ are real values and estimated amounts, respectively; $\overline{P_i}$ is the average of the estimated values, and $n$ is the number of all data. The most accurate model yields respectively 1, 0, and 100 for $R^2$, RMSE, and VAF.

The estimated BC using Z-FIS and FIS compared to the measured one for training and testing parts is displayed in Figures 19 and 20, respectively. As shown, the proposed Z-FIS model presents the highest accuracy for estimating BC as compared to the FIS. The $R^2$ values of 0.977 and 0.971 show the superiority of the FIS model Z-FIS model in estimating the BC. Whilst, the value of 0.912 and 0.904 are achieved for FIS method. Furthermore, the values of other indicators are tabulated in Table 7. The values of RMSE and VAF for training and testing Z-FIS is better than the FIS model. In Table 7, the computational time of these models is also specified. The models were developed in the MATLAB environment and a PC (Intel Core (TM) i3-5010U CPU -2.10 GHz, with 6 GB of RAM, Windows 10).

As shown in Table 7, the computational time for Z-FIS was 18.65 s; while, this value for FIS model was 159.98 s. Therefore, the proposed approach not only decreased the computational time (89.28%) but also increased accuracy. It can be concluded that the proposed model outperforms the FIS method in estimating the BC.

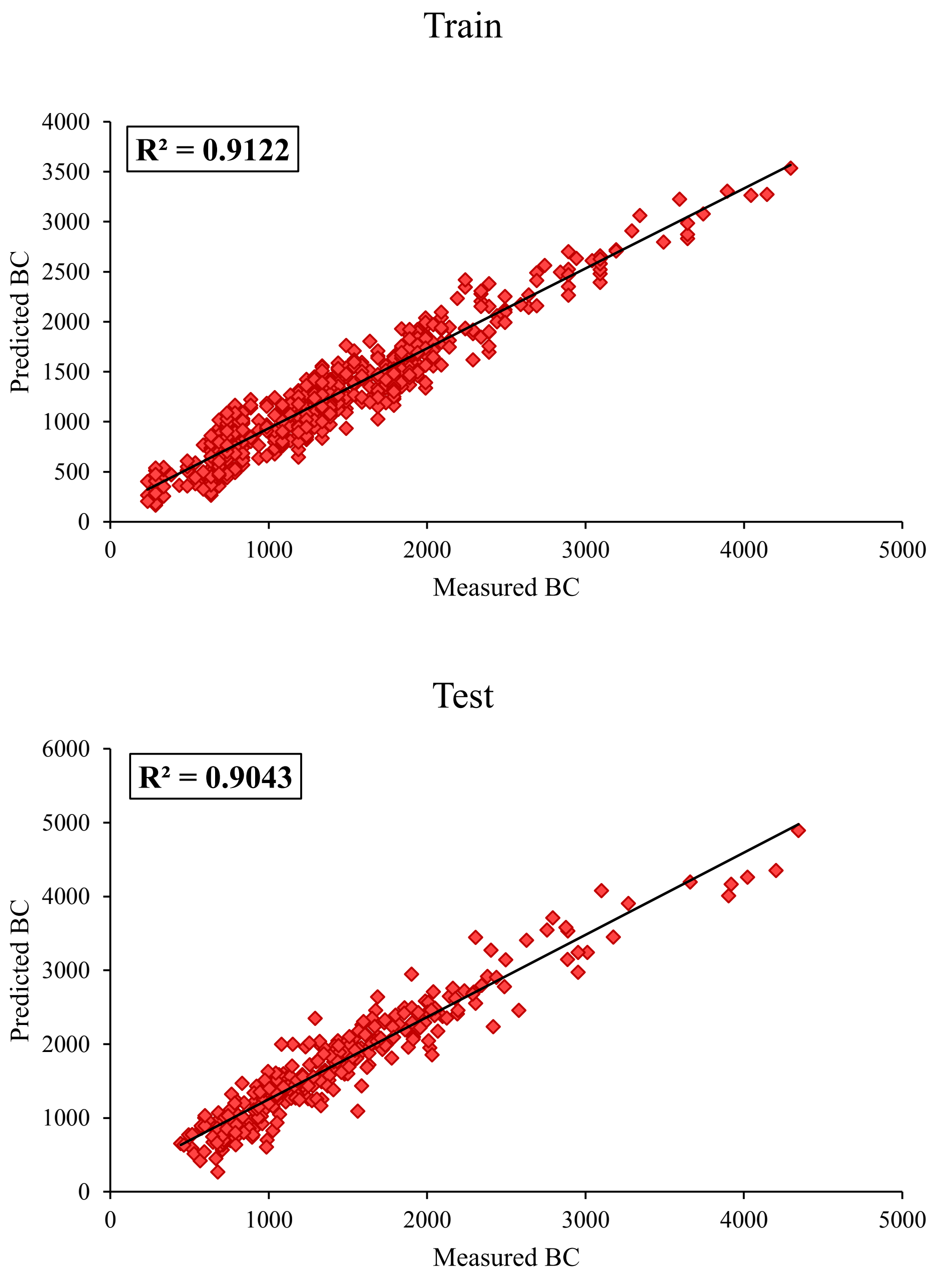
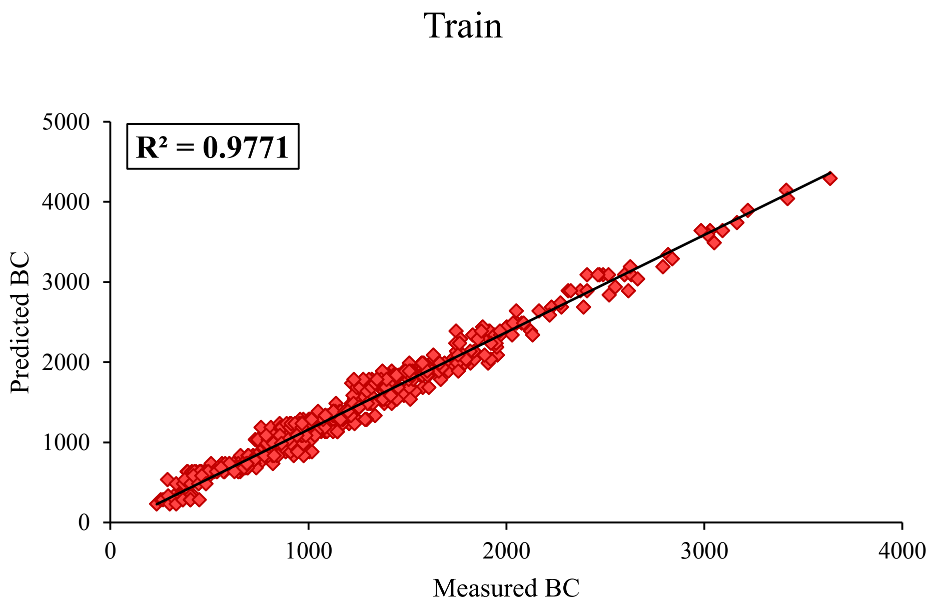



Figure 19. Correlation between measured and predicted BC in training (above) and testing (below) Z-FIS.



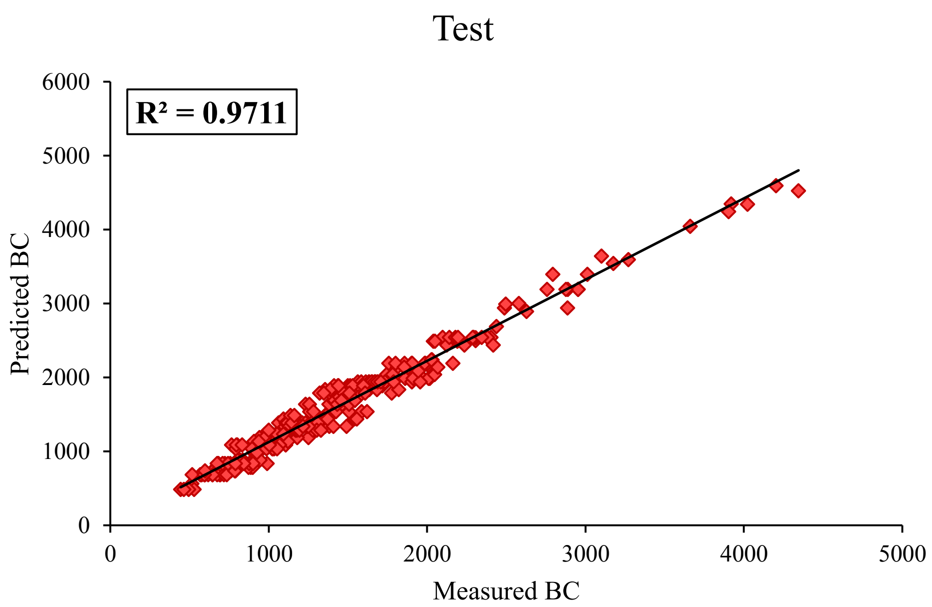




Figure 20. Correlation between measured and predicted BC in training (above) and testing (below) FIS.

Table 7. Performance indices of the predictive models for all datasets

| Predictive model | Performance indices | | | | | | Computational Time (s) |
|---|---|---|---|---|---|---|---|
| | Train | | | Testing | | | |
| | $R^2$ | RMSE | VAF | $R^2$ | RMSE | VAF | |
| FIS | 0.912 | 5.962 | 90.118 | 0.904 | 6.76 | 88.493 | 174.04 |
| Z-FIS | 0.977 | 1.645 | 98.549 | 0.971 | 1.745 | 98.138 | 18.65 |

## 6. Conclusions

Uncertainty always poses problems to engineering projects. Artificial intelligence methods that involve expert opinions have associated with reliability. The fuzzy inference system (FIS) is one of the methods in which the fuzzy rules used are determined based on expert opinions. Therefore, it is obvious that there is uncertainty in it. This paper presents a reliability-based FIS model to predict bearing capacity (BC) based on Z-number concept in civil projects by accounting for uncertainties. In this regard, 968 BC data points were measured, and the most effective independent parameters of estimations were identified. These parameters are depth, density of soil, internal angle of friction, cohesion of soil, and foundation radius. A multiple regression model was constructed to establish relationships between such parameters and the BC values. The obtained results of the proposed model were compared to conventional FIS. The predictive models were constituted by utilising five input parameters (i.e., deep, density of soil, internal angle of friction, cohesion of soil, and foundation radius) to predict BC. It is shown that the Z-FIS model performance is significantly better than the FIS model with the $R^2$ of 0.977 and 0.971 for training and testing part, rspectively. A sensitivity analysis showed that the angle of friction has the most effect on the BC estimations.

## Credit Authorship Contribution Statement

Shahab Hosseini: Conceptualization, methodology, writing—original draft, formal analysis, visualization.

Behrouz Gordan: Writing—original draft, review and editing.

Danial Jahed Armaghani: Supervision, review and editing.

Erol Kalkan: Review and editing.

## Declaration of Competing Interest

Not applicable

## Conflict of interests

This manuscript has not been published or presented elsewhere in part or in entirety and is not under consideration by another journal. There are no conflicts of interest to declare.

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