ACI STRUCTURAL JOURNAL TECHNICAL PAPER

Title no. 103-S38

Queries: 366, 371

Finite Element Analysis and Practical Modeling of Reinforced Concrete Multi-Bin Circular Silos

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Stress resultants in overlapping wall regions (intersection walls) of multi-bin circular silos require a significant computational effort to determine forces due to structural continuity. This paper presents a practical equivalent beam model for computing design forces along the silo walls when subjected to various internal and interstice loadings. The equivalent beam model of intersection wall was developed based on the effective length concept, and verified in a comprehensive series of finite element (FE) analyses of a cluster of four silos for various silo-wall thicknesses. The influence of wall thickness on hoop forces and bending moments acting on interstice and external walls were also examined, and simple empirical expressions were presented for design applications. The proposed beam model yields an accurate estimation of bending moments and hoop forces with a maximum 7% deviation compared with those obtained from detailed FE models.

Keywords: bending moment; force; loading.

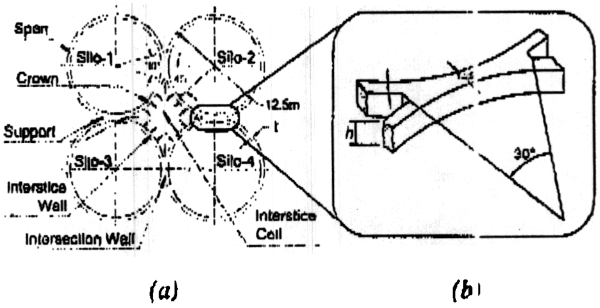

Fig. 1—(a) Typical cluster of four silos; and (b) perspective view to intersection wall.

INTRODUCTION

Multi-bin reinforced concrete (RC) silos are commonly used in industry for storing granular materials within cells and interstice bins. A typical four-silo cluster is shown in Fig. 1. The transition region of two adjacent silos is referred to as intersection-wall, and the wall of the interstice cell is referred to as interstice-wall. The midspan of the interstice-wall is henceforth called the crown, and the external-wall stands for the portion of silo wall other than the interstice and intersection wall. Also shown in Fig. 1(b) is a typical proportion of the intersection wall between two adjacent silos.

Clearly, the grouped silo behavior is different than single silo behavior due to the force transfer in transition regions of neighboring silos. Design standards such as ACI 313-97[1] mentions the effects of loaded and unloaded cell combinations in multi-bin configurations, and the bending moments caused by the continuity of the transition region, but it provides no guidelines to the designer. Therefore, in common practice, structural continuity of the walls along two adjacent silos, which causes horizontal bending moments, is usually ignored due to significant computational effort. On the other hand, bending moments acting on interstice walls, caused by structural continuity when combined with membrane tension, frequently cause cracking, and not providing sufficient horizontal reinforcement to resist such combined effects could lead to a loss of primary strength of the walls.[2] Therefore, detailing of reinforcement in these regions requires accurate computation of design forces. Various methodologies exist in the literature for computing bending moments and hoop forces (that is, membrane forces) considering structural continuity; however, they appear to give a wide scatter of results.[3] For that reason their critical evaluation is essential to provide definite guidelines to the designer.

RESEARCH SIGNIFICANCE

The computation of design forces due to structural continuity in the multi-bin silos is a time-consuming process because each loading combination including loaded and unloaded cells, as well as interstice loadings, should be considered to determine the worst-case loading scenario. Six critical loading combinations (five internal loading cases plus one interstice loading case) should be evaluated for the design of a four-silo cluster. The existing methods proposed in the literature to resolve this problem have their own inherent difficulties in their applications (discussed in more detail in the following sections). Therefore, the objective of this study is to propose a simpler model to compute design forces that can be applicable to the design of interstice and external walls in multi-bin silos. Due to a lack of experimental data, validation of the model is achieved through a comprehensive series of finite element (FE) analyses taking into account various FE modeling approaches as well as silo geometrical configurations. Wall thickness is varied in an extensive parametric study to demonstrate its influence on resultant bending moments and hoop forces. The loading of silo walls is considered as lateral pressure due to interstice loading (interstice cell is filled and cylindrical cells are emptied) and internal loading (combinations of loaded and unloaded cylindrical cells). Results of this study further facilitate the development of empirical coefficients for practical computation of the maximum stress resultants at the external walls and interstice walls.

ACI Structural Journal, V. 103, No. 3, May-June 2006.
MS No. 04-218 received July 12, 2004, and reviewed under Institute publication policies.

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BENCHMARK FINITE ELEMENT MODEL

In this study, an FE model of a typical four-silo cluster (Fig. 1) is used as the baseline comparison. The geometric configuration of the model was previously studied by many researchers,[2-5] and therefore selected herein to allow a direct comparison. The created FE model represents a horizontal strip of unit height in the silo axial direction. The assumption of plain strain accounts for the interaction with adjacent silo strips, because in the model, zero axial displacement was a constraint at all nodes. Use of a strip model in lieu of the complete three-dimensional model is based on the studies that the strip models produce satisfactory results for horizontal membrane tension (that is, hoop force) and bending moment at the pressure zone locations of greatest interest in silo design (that is, regions close to the top and bottom boundaries).[2] Even for very short silos, studies by Prato and Godoy[4] show that a strip model can be used for design purposes with the advantage over general FE assemblies of significantly reduced computational effort.

The modeling of multi-bin silos becomes complicated particularly in the region of common walls (intersection) where the silo walls overlap due to structural continuity (Fig. 1(b)). Initially in this study, three-dimensional solid elements were implemented (that is, eight-noded brick elements) in the FE modeling without accommodating any simplifying assumptions about the geometry, stiffness, and boundary conditions. The FE model was studied to get benchmark results to be used later for the development of the practical beam model. This computer model of the group of four silos together with the FE discretization of intersection walls are shown in Fig. 2. The nodes at the midsection of the intersection wall were allowed to displace only in the longitudinal direction of the intersection wall, and displacements of all the nodes in the vertical direction were restrained. The elastic modulus of concrete was taken as 3.02×10^6 ton/m^2 (29.6 GPa). For design purposes, it is usually assumed that normal pressure acting on the walls is constant at a given elevation. Therefore, the model was analyzed under interstice and internal loading as the uniformly distributed horizontal pressure of 11 ton/m^2 (108 kPa). According to interstice loading (internal loading results are explained later), the results of analyses showed that the critical sections in which the stress resultant reached maximum are the crown of the interstice walls and support region (refer to Fig. 1). The maximum stress distribution in these regions is exhibited in Fig. 3(a). Note that the stress resultants due to interstice loading on the face of the solid element have been computed from the element nodal forces[6] as described in Fig. 3(b). These results will be used in the forthcoming for the validation of the proposed beam model along with comparisons with other simplified FE models, and also results from other studies.

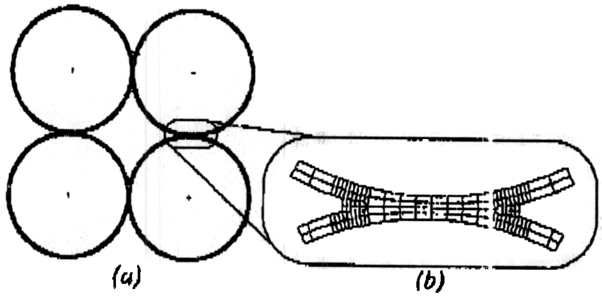

Fig. 2—(a) FE discretization of silo walls using solid elements; and (b) a close-up to intersection wall mesh.

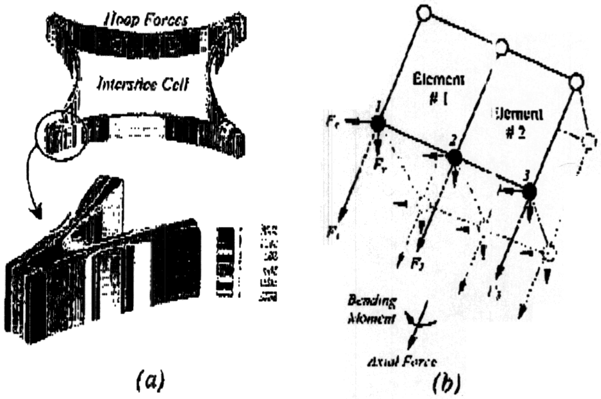

Fig. 3—(a) Stress distribution along transverse (horizontal) direction of interstice walls (unit are in ton/m^2); and (b) computation of stress resultants from nodal forces of solid elements.

PRACTICAL BEAM MODEL DEVELOPMENT

Despite the fact that FE modeling of group circular silos using three-dimensional solid elements produces more accurate results than any other simplified modeling approaches, its application in practice is computationally demanding due to difficulties in mesh generation in overlapping regions and interpretation of nodal forces under various combinations of loaded and unloaded silo cells. To avoid such difficulties, various alternative FE modeling approaches were proposed: Horowitz and Nogueira[7] proposed a mixed element model that used solid elements for intersection walls and shell elements for silo walls (Fig. 4(a)); Stalnaker and Harris[2] used a shell element model (having bending and membrane capabilities) for silo walls and for interstice walls (Fig. 4(b)).

In the shell element model,[2] the overlapping region is modeled with rigid link elements (that is, using shell elements) that may overestimate the stiffness of this region. The main advantage of the method, however, is the reduction in computation effort (decreased number of D.O.F.'s) and ease in result interpretation. A mixed element model[7] seems to reduce the computational effort by decreasing the number of elements and keeping the modeling of the intersection region as accurate as possible. Yet, it may clearly produce unrealistic stress concentrations at transition regions of shell elements to solid elements. Additionally, the difficulties in the modeling of overlapping region and the computation of

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design forces from the stress resultants of solid elements have not been resolved in the mixed element model.

Because of these underlined reasons, we have considered beam model instead of the complicated geometry of the overlapping region. At first glance, a tapered beam element having variable cross section might be used effectively as an alternative to existing FE models. The analysis based on a variable cross section of beam element, however, is still not suitable for practical applications due to similar complexities exiting in the solid and shell FE models. Previous studies show that variable section properties can be well estimated with equivalent beam elements using the effective length concept and average cross section properties.[8] Based on this concept, different geometrical configurations of beam elements were investigated by dividing the intersection wall into a number of beam segments along the centroidal axis of the wall in the longitudinal direction. Each beam segment was modeled with a two-noded beam element having average cross section properties (that is, cross section moment of inertia and area). The equivalent length of the transition beam element was varied from 15 to 25% of the intersection wall (that is, for Element Type-2 in Fig. 5). The transition beam element having 15% length of the intersection wall was found to give the most accurate results when compared with a solid FE model. Therefore, the beam configuration given in Fig. 5 is proposed for the modeling of transition region of multi-bin circular silos, and its validity is discussed in the next section. It should also be noted that the external and interstice walls were also modeled using two-noded beam elements. Because these regions have constant cross section properties, simple beam elements were used without necessitating any complication as in overlapping regions.

COMPARISONS OF BEAM MODEL WITH OTHER FE MODELS

The FE model using the beam element is compared with the shell element model[2] and solid element model (assumed as the most accurate) under the effects of internal and interstice loading conditions. A similar geometrical configuration given in Fig. 1 is used for all cases. Notably, the mixed-element model[7] does not seem to avoid the existing difficulties in the solid model in computation of resultant design forces and moments at interstice and external walls, therefore it is not taken into evaluation.

FE models were analyzed considering various D/t values from 31.25 to 62.50. Based on the interstice loading, the resulting bending moments and hoop forces at the crown and support of the interstice walls are compared in Fig. 6 for the solid model, shell element model, and proposed beam model. An excellent match was obtained for both the hoop forces and bending moments between the solid model and the beam model. In general, the beam model is superior to the shell model by producing hoop forces and bending moments close to benchmark results of the solid model. Differences between the beam and shell models become much clearer for bending moments: the shell model gives smaller bending moments at the crown but yields larger negative moments at the support of the interstice walls.

In Fig. 7, bending moments and hoop forces along the external walls for three of the models are compared for internal loading as the loaded and unloaded silo cell configurations. For the internal loading of the four-silo cluster, five load combinations were examined, and the critical internal load case was obtained when a single cylindrical cell

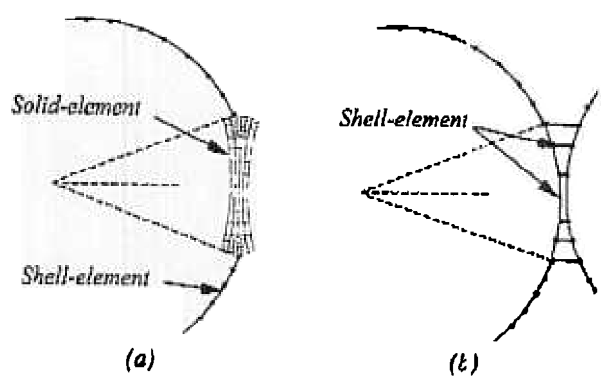
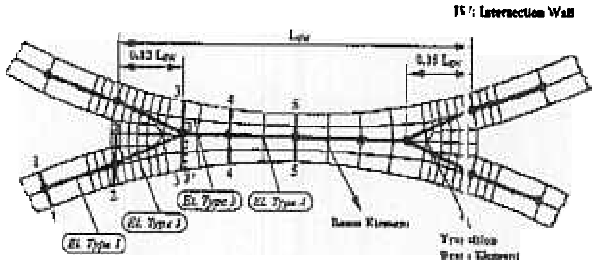


Fig. 4—Finite element simplified models: (a) mixed model;[7] and (b) shell-element model.[2]



Beam Element Section Properties

Element Type #1	Element Type #2	Element Type #3	Element Type #4
$I = I_{1\text{-}1}$ $A = A_{1\text{-}1}$	$I = (I_{2\text{-}2} + I_{3\text{-}3})/2$ $A = (A_{2\text{-}2} + A_{3\text{-}3})/2$	$I = (I_{3\text{-}3} + I_{4\text{-}4})/2$ $A = (A_{3\text{-}3} + A_{4\text{-}4})/2$	$I = (I_{4\text{-}4} + I_{5\text{-}5})/2$ $A = (A_{4\text{-}4} + A_{5\text{-}5})/2$
$I_{i\text{-}i}$, $A_{i\text{-}i}$: Section moment of inertia and area at designated cross sections			

Fig. 5—Practical beam model for modeling of intersection wall.

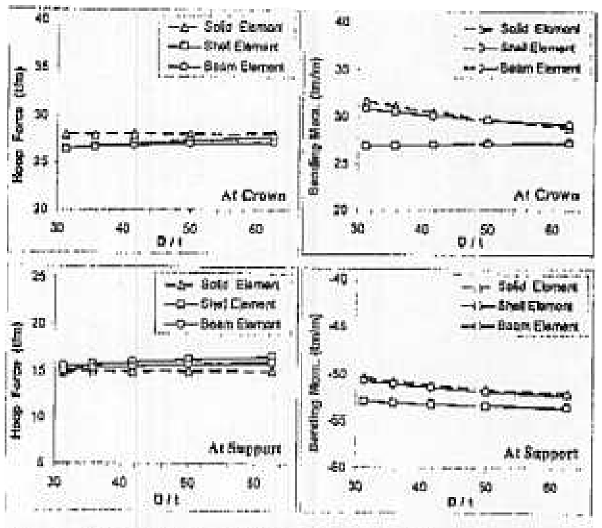


Fig. 6—Comparisons of hoop forces and bending moments at crown and support of interstice wall under interstice loading from models based on beam element, shell element, and solid element for various D/t values.

was filled and the others cells were kept empty. These loading combinations are illustrated in Fig. 8. The results presented in Fig. 7 reflect the numerical projection of a single cell-loading along the external wall. Similar to interstice loading, the beam model gives better estimation of design

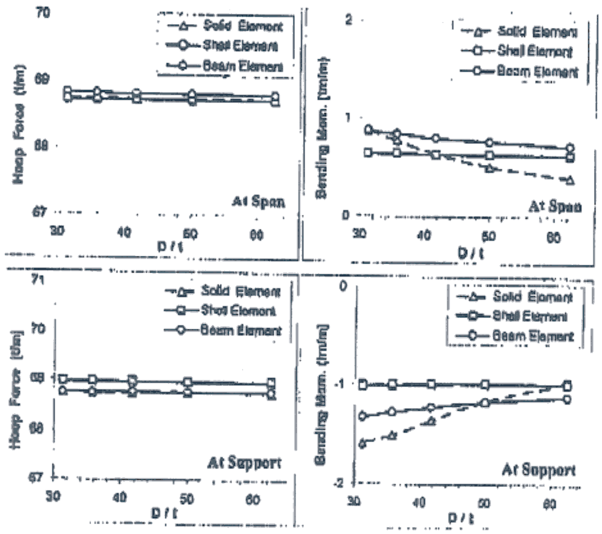


Fig. 7—Comparisons of hoop forces and bending moments at span and support of external wall under internal loading from models based on beam element, shell element, and solid element for various D/t *values.*

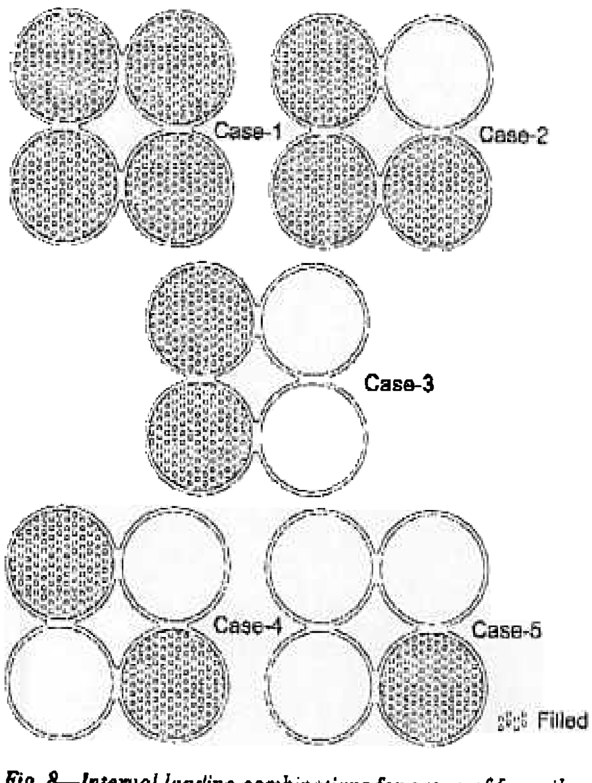


Fig. 8—Internal loading combinations for group of four silos.

forces compared with the shell element model relative to the solid element model. It is also noteworthy that the resultant bending moments along the external wall due to internal loading are negligibly small. However, the critical value of the hoop force, which is an essential design parameter for reinforcement of the silo walls, becomes larger due to the internal loading compared with the interstice loading (refer to Fig. 6). This is basically due to the membrane action of the external walls when subjected to internal loading condition.

Table 1—Comparison of axial force and bending moment computed using various models

	Support		Crown	
	Axial, kN (ton)	Moment, kN·m (ton·m)	Axial, kN (ton)	Moment, kN·m (ton·m)
Safarian and Harris[9]	678.9 (69.2)	1.9 (0.2)	678.9 (69.2)	10.8 (1.1)
	0.0 (0.0)	–1174.8 (–119.8)	285.7 (29.1)	605.2 (61.7)
	153.7 (15.7)	–542.9 (–55.3)	284.0 (29.0)	271.5 (27.7)
	185.4 (18.9)	–484.6 (–49.4)	248.2 (25.3)	302.6 (30.8)
	186.9 (19.1)	–438.8 (–44.7)	249.7 (25.5)	349.8 (35.7)
	148.3 (15.1)	–502.7 (–51.2)	279.3 (28.5)	316.3 (32.2)
	156.8 (16.0)	–507.2 (–51.7)	263.5 (26.9)	308.9 (31.5)

Note: $r = -110$ kN/m^2; $D = 12.5$ m.

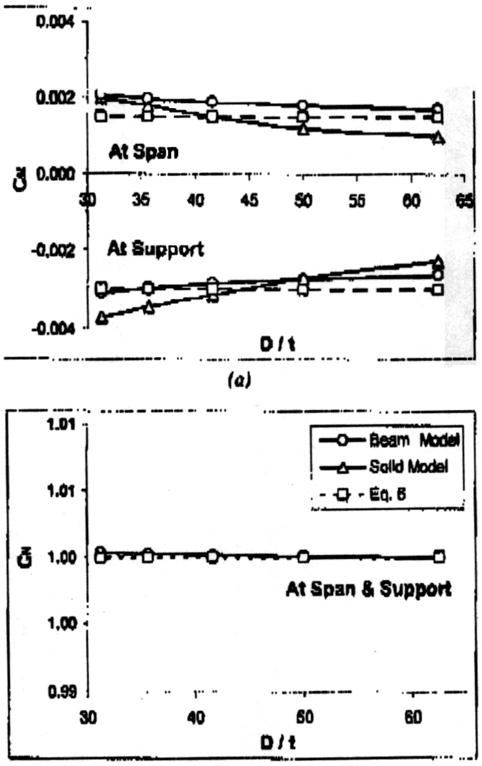
The outward pressure in the cells tends to create a significant tension in the axial direction of the silo wall, and therefore the resultant bending moments become negligible. On the other hand, interstice loading creates an arching effect at the interstice walls that results in considerable bending moments as well as axial thrust at the crown and support of the interstice wall. However, the axial thrust during interstice loading case becomes smaller than that of internal loading.

The results based on the beam model deviate from those obtained from the solid FE model by a maximum of 7%. For the quantification of the deviation, bending moments due to internal loading were not considered because they were negligibly small. Despite the pros and cons of all models from the modeling and computational points, all of the models can be used for design purposes; however the beam model appears to be the easiest to analyze and interpret while retaining the desirable engineering accuracy.

The results obtained from the practical beam model are further compared in Table 1 with other modeling approaches. The method by Timm and Mindels,[9] which assumes that free axial force and hoop displacement at wall supports, produced the largest moment values at both support and crown. Conversely, bending moments through FE solutions of Safarian and Harris[10] are unrealistically low due to their modeling assumption of restrained wall against axial or hoop movement. The method by Ciesielski[11] yields closer results to those of solid FE model and practical beam model in terms of both axial forces and bending moments. Similarly, results obtained by the method of Hayd [3] are also comparable with findings of this study. These more realistic results of Ciesielski and Haydl stem from their modeling assumption that wall supports have partial restraints from the attached silos.

EMPIRICAL DESIGN FORCE COEFFICIENTS

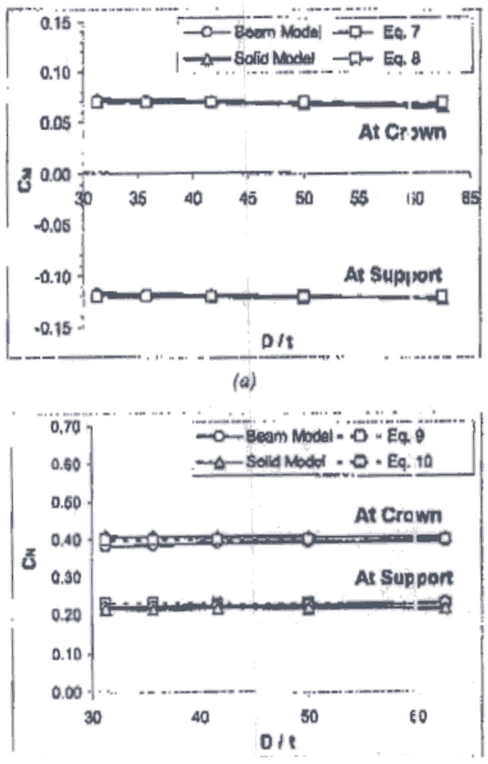
In the previous sections, the beam-model was introduced for the discretization of complex geometry of overlapping region as well as the modeling of interstice and external walls. To complete the practical framework of the study, alternative empirical design formulas are introduced here to compute the bending moments, hoop forces at both interstice and external walls, and shear forces at interstice walls without necessitating finite element discretization.



(a)

(b)

Fig. 9—(a) Positive and negative moment coefficients; and (b) hoop force coefficients, obtained from beam model, solid model, and from simplified equations for span and support of external wall in case of internal loading. (Note: Eq. (4) for moment at span; Eq. (5) for moment at support.)



(a)

(b)

Fig. 10—(a) Positive and negative moment coefficients; and (b) hoop force coefficients, obtained from beam model, solid model, and from simplified equations for span and support of interstice wall in case of interstice loading. (Note: Eq. (7) and (9) for crown; Eq. (8) and (10) for support moments and hoop forces, respectively.)

Studies by Stalnaker and Harris[2] show that the bending moment due to structural continuity in multi-bin circular silos can be estimated by a simple relationship of radius, pressure and a coefficient obtained from the FE analyses. Their equation has the following form

$$M = C_M \cdot p \cdot r^2 \tag{1}$$

where M is the moment per unit height of the silo wall; p is the pressure applied due to stored material; and r is the silo radius. C_M is the estimator coefficient deduced from the FE analyses for positive and negative values of stress resultants. It should be noted that Stalnaker and Harris[2] investigated the internal loading conditions only and derived the bending moment coefficient of C_M based on their FE analyses on four and six silo clusters (recall that their FE models were based on shell elements).

For the completeness of this approach, the following two equations are developed here as the modified version of Eq. (1) to estimate the hoop forces along the silo walls and shear forces at interstice walls

$$N = C_N \cdot p \cdot r \tag{2}$$

$$N = C_V \cdot p \cdot r \tag{3}$$

where N and V stand for the axial hoop force and shear force, respectively. C_N and C_V are the point estimators found using the solid model and practical beam model. The FE models created using solid elements and beam elements were reanalyzed for silo wall thicknesses of 0.20, 0.25, 0.30, 0.35, and 0.40 m with a constant silo diameter of 12.50 m and internal lateral pressure of 11 ton/m^2 (108 kPa). The analyses were repeated for interstice loading as well as five internal load cases accounting for the empty and loaded cell combinations. For each case, output of the FE models was scanned for the largest positive and negative bending moment coefficients, hoop force coefficients for the external and interstice walls, as well as shear force coefficient for interstice walls.

It should be noted that positive moment causes tensile flexural stress on the inside surface of the wall. Fig. 9 and 10 exhibit the influence of D/t variation on the estimator coefficients of C_M and C_N for external walls under internal loading and interstice walls under interstice loading conditions, respectively. The coefficients can be considered as reasonable approximations for a range of common D/t values (wall stiffness) from 31.25 to 62.50. Also noteworthy is that the solid model and the beam model yielded consistent results. C_N is found to be insensitive to D/t for all loading cases. Therefore, it can be well approximated using Eq. (2) by taking the C_N value as 1.0 for external walls. In case of interstice walls, however, different C_N coefficients are essential for the support and crown hoop forces, therefore use of a constant C_N as 0.23 for support and 0.40 for crown may produce satisfactory results.

Table 2—Comparison of critical shear force at interstice walls

Interstice wall thickness t, m	Solid FE model, kN (ton)	Practical beam model, kN (ton)
0.20	484.6 (49.40)	
	484.4 (49.38)	
	483.8 (49.32)	
	483.3 (49.26)	
	482.7 (49.20)	

A bending moment coefficient of C_M shows more variation with respect to D/t. C_M becomes larger as D/t value becomes smaller because the walls become stiffer. The critical loading case for interstice walls is observed to be the interstice loading; and for external walls, it is the internal loading. Under this loading condition, the critical shear force may occur at the support of the interstice walls, whereas it becomes negligible at the span. Table 2 compares the shear force variation computed based on a solid FE model and proposed beam model. Whereas slight shear force variation is observed with the change in interstice wall thickness, in general, both modeling alternatives give analogous results, which can be approximated with a constant C_V value of 0.67.

These results suggest that the generalized forces at interstice and external walls can be computed for practical applications from the following set of equations for the span and support of the external walls as well as crown and support of the interstice walls. It is also worth mentioning that the empirical coefficients are unit dependent, and consistent units given in this paper should be used for their applications.

For external walls

$$M_{SPAN} = (0.0015) \cdot p \cdot r^2 \tag{4}$$

$$M_{SUPPORT} = (-0.0030) \cdot p \cdot r^2 \tag{5}$$

$$N_{SUPPORT} = N_{SPAN} = (1.0) \cdot p \cdot r \tag{6}$$

For interstice walls

$$M_{CROWN} = (0.07) \cdot p \cdot r^2 \tag{7}$$

$$M_{SUPPORT} = (-0.12) \cdot p \cdot r^2 \tag{8}$$

$$N_{CROWN} = (0.40) \cdot p \cdot r \tag{9}$$

$$N_{SUPPORT} = (0.23) \cdot p \cdot r \tag{10}$$

$$V_{SUPPORT} = (0.67) \cdot p \cdot r \tag{11}$$

CONCLUSIONS

Several available FE models for computing the maximum bending moments and hoop forces along the walls of multi-bin circular silos have been examined. Major difficulties in these models are the mesh generation due to continuity of the silo walls in the overlapping region, the significant computational effort due to size of the mesh (that is, the large number of DOF), as well as the conversion of nodal outputs into hoop forces and bending moments. To minimize the complexities of current modeling approaches, a practical beam model is proposed in this paper. The model was verified through a comprehensive parametric study against benchmark results of FE models having three-dimensional solid mesh. Analysis of results shows that the proposed model is simple and accurate enough to compute the hoop forces, shear forces, and bending moments along the external and interstice walls of multi-bin circular silos to be used for design.

In the proposed model, two-dimensional beam elements are used for modeling the interstice and external walls. For modeling the intersection walls, average sectional properties are used. The transition region at the intersection wall is modeled using beam elements having a length of 15% of the total intersection wall length. The results of the proposed beam model yield maximum deviation of 7% compared with solid FE element model.

It should also be pointed out that the results presented here do not reflect the special variation of stresses, because they are derived from a plane strain model of the silo with no consideration for the variation of internal pressure and discharge pressure with height, as well as no consideration for the global restraining effects of the base and top. Also, uniform pressure is assumed in the horizontal plane.

Free sway at the top due to wind or seismic forces and global restraining effects at the base require special consideration regarding the approximation by using the beam model that is not discussed within the scope of this paper. Numerical results cited are only used to illustrate the applicability of the practical beam model for modeling of interstice, intersection, and external walls. Common values of geometric parameters were considered for the model verification. Therefore, the presented results are valid only for the four-silo configuration having the same diameter cells, but the approach can easily be expanded to cover other silo clusters.

Numerical projections based on the simple beam models are sufficiently accurate in illustrating the influence of D/t on bending and in-plane stress resultants for a typical four-silo cluster. The largest bending moment becomes critical for interstice walls under interstice loading whereas hoop forces are more critical for the design of external walls under internal loading cases.

The bending moment and hoop forces on a group of four silos are expressed by simple expressions. Various D/t values in FE models were considered. While the values of C_N and C_M vary slightly with D/t, the same basic pattern is always observed for each case, and the variation is negligible for design purposes. Therefore constant design force coefficients in the empirical equations are proposed. The design coefficients were obtained considering the interstice loading as well as the worst-case load combinations of loaded and unloaded silos under internal loading.

The research reported herein provides more insight in behavior of multi-bin circular silos under several different loading conditions and develops a practical framework by proposing the beam-model as an alternative FE modeling of complex geometry of silo clusters and suggesting empirical equations for practical estimation of interstice and external wall design forces.

NOTATION

A = cross-sectional area
C_M = estimator coefficient for positive and negative values of bending moment
C_N = estimator coefficient for hoop force
D = silo diameter

I	=	moment of inertia of section
M	=	moment per unit height of silo wall
N	=	axial hoop force
p	=	pressure applied by stored material
r	=	silo radius
t	=	wall thickness
V	=	Shear force

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