How Many Records Should Be Used in an ASCE/SEI-7 Ground Motion Scaling Procedure?

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U.S. national building codes refer to the ASCE/SEI-7 provisions for selecting and scaling ground motions for use in nonlinear response history analysis of structures. Because the limiting values for the number of records in the ASCE/SEI-7 are based on engineering experience, this study examines the required number of records statistically, such that the scaled records provide accurate, efficient, and consistent estimates of "true" structural responses. Based on elastic–perfectly plastic and bilinear single-degree-of-freedom systems, the ASCE/SEI-7 scaling procedure is applied to 480 sets of ground motions; the number of records in these sets varies from three to ten. As compared to benchmark responses, it is demonstrated that the ASCE/SEI-7 scaling procedure is conservative if fewer than seven ground motions are employed. Utilizing seven or more randomly selected records provides more accurate estimate of the responses. Selecting records based on their spectral shape and design spectral acceleration increases the accuracy and efficiency of the procedure. [DOI: 10.1193/1.4000066]

INTRODUCTION

When nonlinear response history analysis (RHA) is required for design verification of certain building structures (for example, tall buildings, buildings with damping devices or base isolation, etc.), the International Building Code (ICBO 2009) and California Building Code (ICBO 2012) refer to the ASCE/SEI-7 Section 16.2[1] (ASCE 2005, 2010). According to these documents, earthquake records should be selected from events of magnitudes, fault distance and source mechanisms that are consistent with the maximum considered earthquake (MCE), which is used as the notation of 90% probability of not being exceeded in a certain exposure time period, which is generally 2,475 years.

For two-dimensional analysis of symmetric-plan buildings, ASCE/SEI-7 requires intensity-based scaling of ground motion records using appropriate scale factors so that the mean[2] value of the 5%-damped response spectra for the set of scaled records is not less than the design response spectrum over the period range from $0.2T_n$ to $1.5T_n$ (where T_n is the elastic first-"mode" vibration period of the structure). For three-dimensional analyses,

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[1]Because the ground motion scaling procedure for two-dimensional analysis of structures is same in ASCE/SEI-7-05 and -7-10 documents, we simply refer to this method as the ASCE/SEI-7 scaling procedure in the remaining of this paper.

[2]In this manuscript, "mean" is used in lieu of "arithmetic mean."

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ground motions should consist of pairs of appropriate horizontal ground motion acceleration components. For each pair of horizontal components, a square root of the sum of the squares (SRSS) spectrum should be constructed by taking the SRSS of the 5%-damped response spectra of the unscaled components. Each pair of motions are then scaled with the same scale factor such that the mean of the SRSS spectra from all horizontal component pairs does not fall below the corresponding ordinate of the target spectrum in the period range from $0.2T_n$ to $1.5T_n$. The design value of an engineering demand parameter (EDP)—member forces, member deformations or story drifts—is taken as the mean value of the EDP over seven (or more) ground motions, or its maximum value over all ground motions, if the system is analyzed for fewer than seven ground motions. This procedure requires a minimum of three records. These limits on the number of ground motions are based on engineering experience rather than a comprehensive evaluation (personal communication with Charlie Kircher and Nico Luco).

This study, for the first time, statistically examines the required number of records for the ASCE/SEI-7 procedure such that the scaled records provide accurate, efficient and consistent estimates of "true" median structural responses. The adjective "accurate" refers to the discrepancy between the "true" responses and those computed from the group of scaled records. The adjective "efficient" refers to the record-to-record (i.e., intraset) variability of responses, and the adjective "consistent" refers to the ground motion set-to-set (i.e., interset) variability of accuracy and efficiency. Smaller values of interset and intraset dispersion of responses indicate that the scaling procedure is more efficient and consistent.

Based on elastic-perfectly plastic and bilinear single-degree-of-freedom (SDOF) systems, the accuracy, efficiency and consistency of the ASCE/SEI-7 ground motion scaling procedure are evaluated by applying it to 480 sets of ground motions. For brevity, this paper presents results from bilinear systems only; results for elastic-perfectly plastic systems can be found in Reyes and Kalkan (2011). The number of records in these sets varies from three to ten. The scaled records in each set were selected in three different ways: (i) randomly; (ii) minimizing discrepancy between scaled spectrum of a record and the design spectrum over the period range from $0.2T_n$ to $1.5T_n$ (this approach will be referred to as "Best1"); (iii) minimizing discrepancy between scaled spectrum of a record and the design spectrum over the period range from $0.2T_n$ to $1.5T_n$, and then identifying the final set of records with spectral acceleration values at T_n close to that of the design spectrum (this approach will be referred to as "Best2").

GROUND MOTIONS SELECTED

Thirty records selected for this investigation (listed in Table 1) were recorded from seven shallow crustal earthquakes compatible with the following hazard conditions:

- Moment magnitude: $M_w = 6.7 \pm 0.2$
- Joyner-Boore distance: $R_{JB} = 25 \pm 5$ km
- NEHRP soil type: C or D
- Highest usable period[3] ≥ 4 sec

[3]Highest usable period is the low-cut corner frequency of the bandpass filter applied. Because the highest usable period is greater than 4 sec, records in Table 1 have enough long-period content to compute their spectra up to 3 sec reliably.

Table 1. Selected ground motion records

Record Sequence Number	Earthquake Name	Year	Station Name	Earthquake Magnitude (M_w)	Joyner-Boore Distance (km)	NEHRP Site Class	Highest Usable Period (sec.)
1	San Fernando, Calif.	1971	LA - Hollywood Stor FF	6.6	22.8	D	4
2	San Fernando, Calif.	1971	Santa Felita Dam (Outlet)	6.6	24.7	C	8
3	Imperial Valley (AS), Calif.	1979	Calipatria Fire Station	6.5	23.2	D	8
4	Imperial Valley (AS), Calif.	1979	Delta	6.5	22.0	D	16
5	Imperial Valley (AS), Calif.	1979	El Centro Array #1	6.5	19.8	D	8
6	Imperial Valley (AS), Calif.	1979	El Centro Array #13	6.5	22.0	D	4
7	Imperial Valley (AS), Calif.	1979	Superstition Mtn Camera	6.5	24.6	C	8
8	Irpinia, Italy	1980	Brienza	6.9	22.5	C	4
9	Superstition Hills (AS), Calif.	1987	Wildlife Liquef. Array	6.5	23.9	D	8
10	Loma Prieta, Calif.	1989	Agnews State Hospital	6.9	24.3	D	4
11	Loma Prieta, Calif.	1989	Anderson Dam (Downstream)	6.9	19.9	C	4
12	Loma Prieta, Calif.	1989	Anderson Dam (L Abut)	6.9	19.9	C	8
13	Loma Prieta, Calif.	1989	Coyote Lake Dam (Downst)	6.9	20.4	D	8
14	Loma Prieta, Calif.	1989	Coyote Lake Dam (SW Abut)	6.9	20.0	C	8
15	Loma Prieta, Calif.	1989	Gilroy Array #7	6.9	22.4	D	4
16	Loma Prieta, Calif.	1989	Hollister - SAGO Vault	6.9	29.5	C	8
17	Northridge, Calif.	1994	Castaic - Old Ridge Route	6.7	20.1	C	8
18	Northridge, Calif.	1994	Glendale - Las Palmas	6.7	21.6	C	6
19	Northridge, Calif.	1994	LA - Baldwin Hills	6.7	23.5	D	6
20	Northridge, Calif.	1994	LA - Centinela St	6.7	20.4	D	4
21	Northridge, Calif.	1994	LA - Cypress Ave	6.7	29.0	C	4
22	Northridge, Calif.	1994	LA - Fletcher Dr	6.7	25.7	C	5
23	Northridge, Calif.	1994	LA - N Westmoreland	6.7	23.4	D	4
24	Northridge, Calif.	1994	LA - Pico & Sentous	6.7	27.8	D	5
25	Kobe, Japan	1995	Abeno	6.9	24.9	D	16
26	Kobe, Japan	1995	Kakogawa	6.9	22.5	D	8
27	Kobe, Japan	1995	Morigawachi	6.9	24.8	D	10
28	Kobe, Japan	1995	OSAJ	6.9	21.4	D	16
29	Kobe, Japan	1995	Sakai	6.9	28.1	D	8
30	Kobe, Japan	1995	Yae	6.9	27.8	D	16

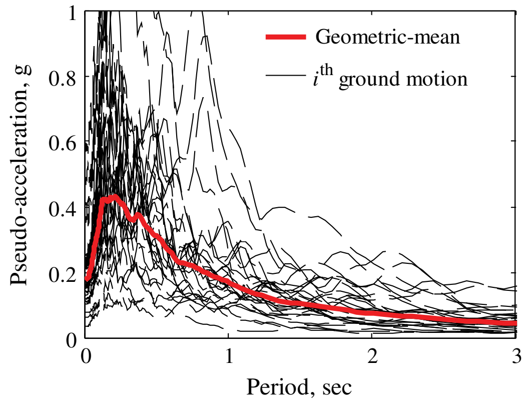

Figure 1. Response spectra of thirty ground motions and their geometric-mean used as the target (that is, "design") spectrum. Damping ratio 5%.

Shown in Figure 1 are the 5%-damped geometric-mean response spectra for the x-component (randomly selected from the two horizontal components) of the unscaled ground motions. The geometric-mean spectrum of thirty records is taken as the design spectrum (that is, target spectrum) for purposes of this investigation.

DESCRIPTION OF INELASTIC SDOF SYSTEMS

The structures considered are 16 SDOF systems with vibration periods equal to 0.2, 0.5, 1, and 2.5 sec, and yield strength reduction factors R equal to 1, 2, 4, and 8. The design base shear is determined as the mass of the system (assumed to be 1 kip-sec^2/in) times the geometric-mean pseudo-acceleration at T_n divided by R. The damping ratio of the selected SDOF systems is 5%. The two constitutive models used for the inelastic SDOF systems are: (1) an elastic–perfectly plastic model, and (2) a bilinear model with 10% strength hardening ratio. In this paper, results for elastic-perfectly plastic systems are not included, but commentaries about these analyses are covered.

METHODOLOGY

According to the ASCE/SEI-7 procedure for two-dimensional (or planar) analyses of "regular" structures, the ground motions should be scaled such that the mean value of the 5%-damped response spectra for the set of scaled motions is not less than the design spectrum over the period range from $0.2T_n$ to $1.5T_n$. The ASCE/SEI-7 scaling procedure does not insure a unique scaling factor for each record; obviously, various combinations of scaling factors can be defined to insure that the mean spectrum of scaled records remains above the target spectrum over the specified period range. To achieve the desirable goal of scaling each record with a minimum scale factor closest to unity, we implemented the ASCE/SEI-7 scaling procedure for randomly selected ground motions as in the following:

1. For each of the thirty records listed in Table 1, calculate the 5%-damped response spectrum $A(T)$ and the vector $\mathbf{A}$ of spectral values at 300 logarithmically spaced periods T over the period range from $0.2T_n$ to $1.5T_n$.
2. Obtain a target (that is, "design") pseudo-acceleration spectrum $\hat{A}(T)$ as the geometric-mean spectrum of thirty records. Define $\hat{\mathbf{A}}$ as a vector of target spectral values $\hat{A}_i$ at periods T over the period range from $0.2T_n$ to $1.5T_n$.
3. For each record, compute the scaling factor SF_1 to minimize the difference between the target spectrum $\hat{A}(T)$ (Step 2) and the response spectrum $A(T)$ (Step 1) by solving the following minimization problem for each ground motion $\min_{SF_1} \|\log \hat{\mathbf{A}} - SF_1 \times \log \mathbf{A}\| \Rightarrow SF_1$, where $\|\cdot\|$ is the Euclidean norm. Required for this purpose is a numerical method to minimize scalar functions of one variable; such methods are available in textbooks on numerical optimization (for example, Nocedal and Stephen, 2006). This minimization ensures that each scaled response spectrum is as close as possible to the target spectrum, as shown schematically in Figure 2.
4. Randomly select a set of m ground motions to be used in nonlinear RHA of the systems described previously. No more than two records from the same event should be included in a single set, so that no single event is dominant within a set.
5. Determine the vector $\hat{\mathbf{A}}_{\text{scaled}}$ for the mean scaled spectrum defined as the mean of the scaled spectra $(SF_1 \times \mathbf{A})$ of the set of m records. The ordinates of this mean scaled spectrum could be smaller than the ordinates of the target spectrum at the same periods.
6. Calculate the maximum normalized difference $\varepsilon_{\text{ASCE}}$ (Figure 3a) between the target spectrum $\hat{\mathbf{A}}$ and the mean scaled spectrum $\hat{\mathbf{A}}_{\text{scaled}}$ over the period range from $0.2T_n$ to $1.5T_n$; that is, $\varepsilon_{\text{ASCE}} = \max_{0.2T_1 \le T_i \le 1.5T_1} (\hat{A} - \hat{A}_{\text{scaled},i}) \div \hat{A}_i$, where $\hat{A}_i$ and $\hat{A}_{\text{scaled},i}$ are the ordinates of the target and the mean scaled pseudo-acceleration spectra at vibration period T_i, respectively. Define the scale factor $SF_2 = (1 - \varepsilon_{\text{ASCE}})^{-1}$.

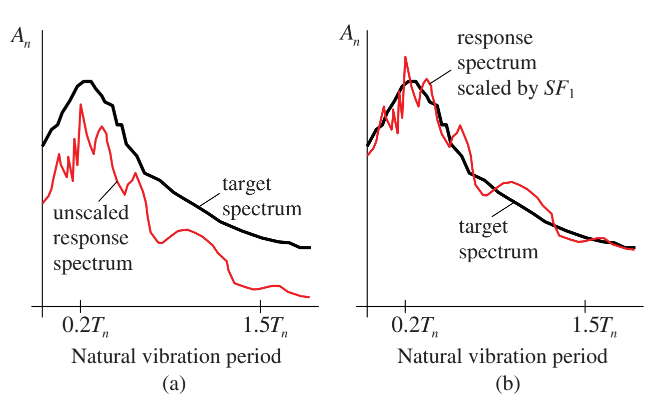

Figure 2. Schematic illustration of Step 3 of the evaluation methodology.

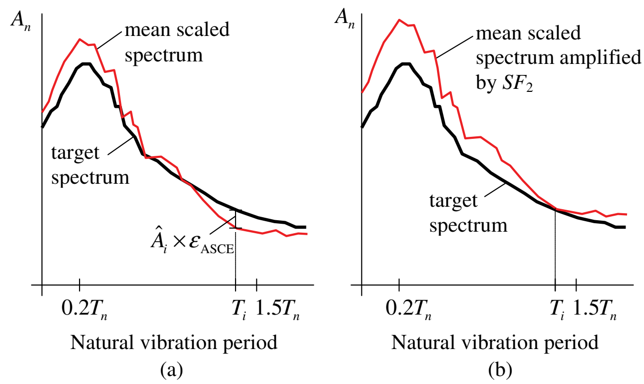

Figure 3. Schematic illustration of Steps 6 and 7 of the evaluation methodology.

7. Determine the final scale factor $SF = SF_1 \times SF_2$ for each ground motion. Scaling ground motions by the scaling factor SF ensures that the mean value of the response spectra for the set of scaled motions is not less than the target spectrum over the period range from $0.2T_n$ to $1.5T_n$ (Figure 3b).

To select ground motions using the approach "Best1," where the discrepancy between the scaled spectrum of a record and the target spectrum over the period range from $0.2T_n$ to $1.5T_n$ is minimized, Step 4 is modified as follows:

4. Rank the scaled records based on their $\|\log\ \hat{\mathbf{A}} - SF_1 \times \log\ \mathbf{A}\|$ value; the record with the lowest value is ranked the highest. From the ranked list, select a set of m records to be used in nonlinear RHA of the systems described previously.

Selection of ground motions using the approach "Best2" requires that Steps 4-7 are iteratively implemented until the quantities $\|\log\ \hat{\mathbf{A}} - SF \times \log\ \mathbf{A}\|$ and $|A(T_n) - \mathrm{SF} \times A(T_n)|$, are minimized. By executing Steps 1 to 7, the scaling factors for the sets of m ground motions would have been determined. Nonlinear RHA is, then, conducted to obtain final EDP values. If at least seven ground motions are analyzed $(m \geq 7)$, the design values of EDPs are taken as the mean or median[4] of the EDPs over the ground motions used. If fewer than seven ground motions are analyzed, the design values of EDPs are taken as the maximum values of the EDPs.

BENCHMARK INELASTIC DEFORMATIONS

Benchmark values, defined as peak inelastic deformations (D_n), were determined by conducting nonlinear RHA of the SDOF systems described previously subjected to each of the 30 unscaled hazard-compatible ground motions, and computing the median and mean value of the data set.

[4]Because the geometric mean and median of a lognormal distribution are the same, we decided to employ the term "median" instead of geometric mean, as is commonly done.

As mentioned in Hancock et al (2008), the empirical ground-motion models that are used to derive the target spectrum assume the ground motions to be lognormally distributed; therefore, the use of the median response spectrum of the records as a target spectrum is more consistent with the specification of the target spectrum. Similarly, it is organic to assume that EDPs are lognormally distributed, when ground motion intensity parameters have the same distribution (Cornell et al. 2002); for this reason, it is more appropriate to represent the "mean" structural response by the median; a conclusion that is widely accepted. However, the ASCE/SEI-7 procedure states that the mean values of EDPs are used if at least seven ground motions are considered. Therefore, we decided to use both the median and the mean of the inelastic deformations as the benchmark values. It should be noted that in all cases of benchmark computations, the mean is larger than the median of inelastic deformations, indicating that the distribution of D_n is positively skewed. The differences between the two (that is, [mean − median] ÷ median) are in the ranges of 15% to 63%, 23% to 39%, 38% to 48%, and 42% to 63% for elastic–perfectly plastic systems with R equal to 1, 2, 4, and 8, respectively. For bilinear systems the differences are 11% to 26%, 12% to 32%, 20% to 45%, and 27% to 56% for the same R values. Note that the difference between mean and median increases with increasing R value.

EVALUATION OF ASCE/SEI-7 SCALING PROCEDURE: FEWER THAN SEVEN GROUND MOTIONS

The ASCE/SEI-7 scaling procedure was implemented for the inelastic SDOF systems of this investigation subjected to one component of ground motion (Table 1). The accuracy of the ASCE/SEI-7 procedure was evaluated first by comparing the maximum value of the inelastic deformation due to seven sets of 3 to 6 scaled records against the benchmark value, defined as the median (or mean) value of D_n due to the 30 unscaled ground motions. These comparisons are shown in Figures 4 and 5 for bilinear systems with $T_n = 0.2$, 0.5, 1, and 2.5 sec, and $R = 1$, 2, 4, and 8 due to groups of 3, 4, 5, and 6 records called as G3, G4, G5, and G6, respectively. Seven sets of records were considered in each of these groups. Among these seven sets, the first five sets of records were selected randomly out of 30 records, and the two remaining sets of records were selected with the criteria explained previously; these are sets "Best1" and "Best2." For each T_n, R, and constitutive model combinations, a total of 30 sets of records are employed. For the benchmark, the blue dot and the vertical line in Figures 4 and 5 represent the median deformation value plus and minus one standard deviation (henceforth denoted as $\pm\sigma$) assuming a lognormal distribution. For each set, the vertical line and the dot represent the range of the data set and maximum deformation value, respectively. Similar plots are presented in Reyes and Kalkan (2011) for elastic-perfectly plastic systems.

Figures 4 and 5 permit the following observations: (1) Increasing the number of records from 3 to 6 has a minor effect in the accuracy of the procedure; overestimations range from 0.4% to 540% and from 35% to 680% for groups G3 and G6, respectively. In the context of this paper, the term "overestimation" means that the mean prediction exceeds the benchmark result. The percentile values were obtained by taking the difference between predicted and benchmark results, and then normalizing it by the benchmark value. (2) The accuracy of the procedure decreases with increasing R value; the maximum error increases from 310% to

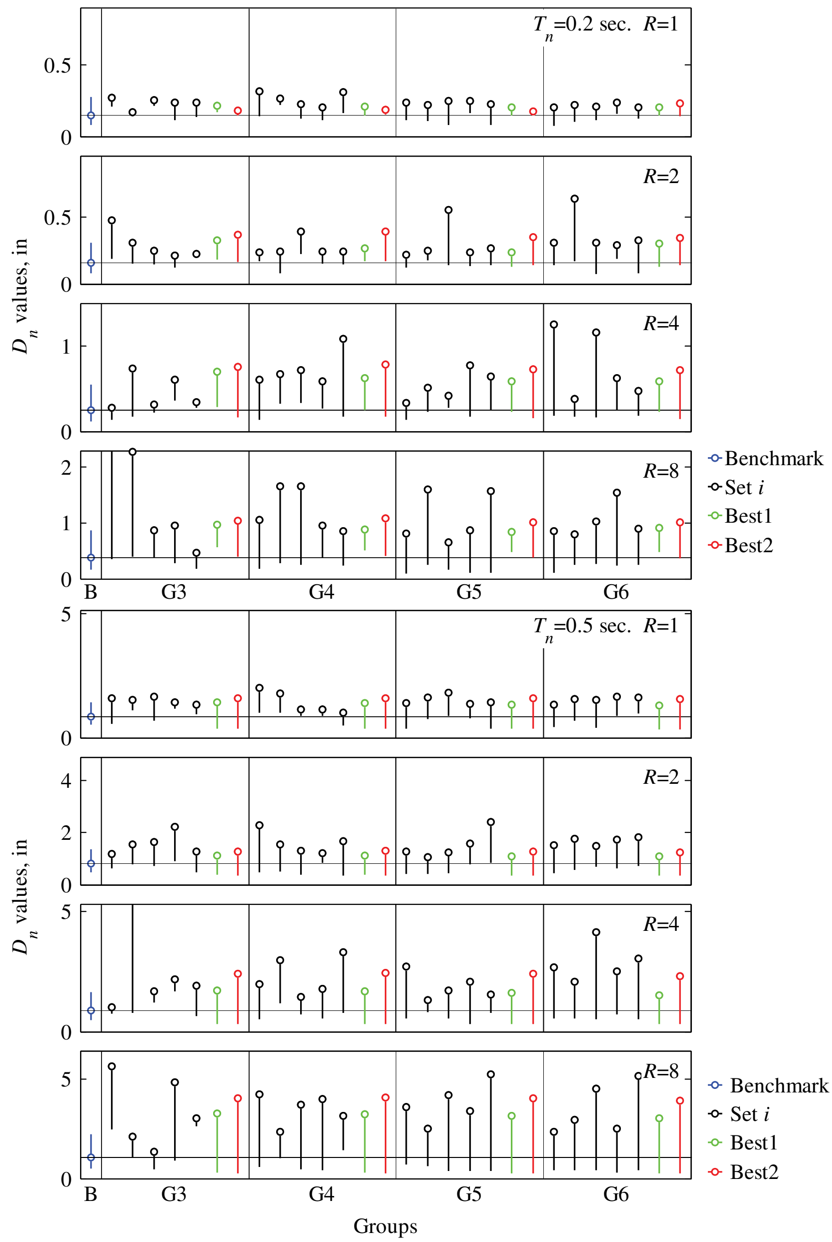

Figure 4. Range of inelastic deformation values for bilinear systems with $T_n = 0.2$ (top panels) and 0.5 sec (bottom panels), and $R = 1$, 2, 4, and 8 for different sets of 3, 4, 5, and 6 ground motion records denoted respectively as G3, G4, G5, and G6. The blue dot and the vertical line represent the benchmark (B) median deformation value $\pm\sigma$ assuming a lognormal distribution. For each set, the vertical line and the dot represent the range of the data set and maximum deformation value, respectively.

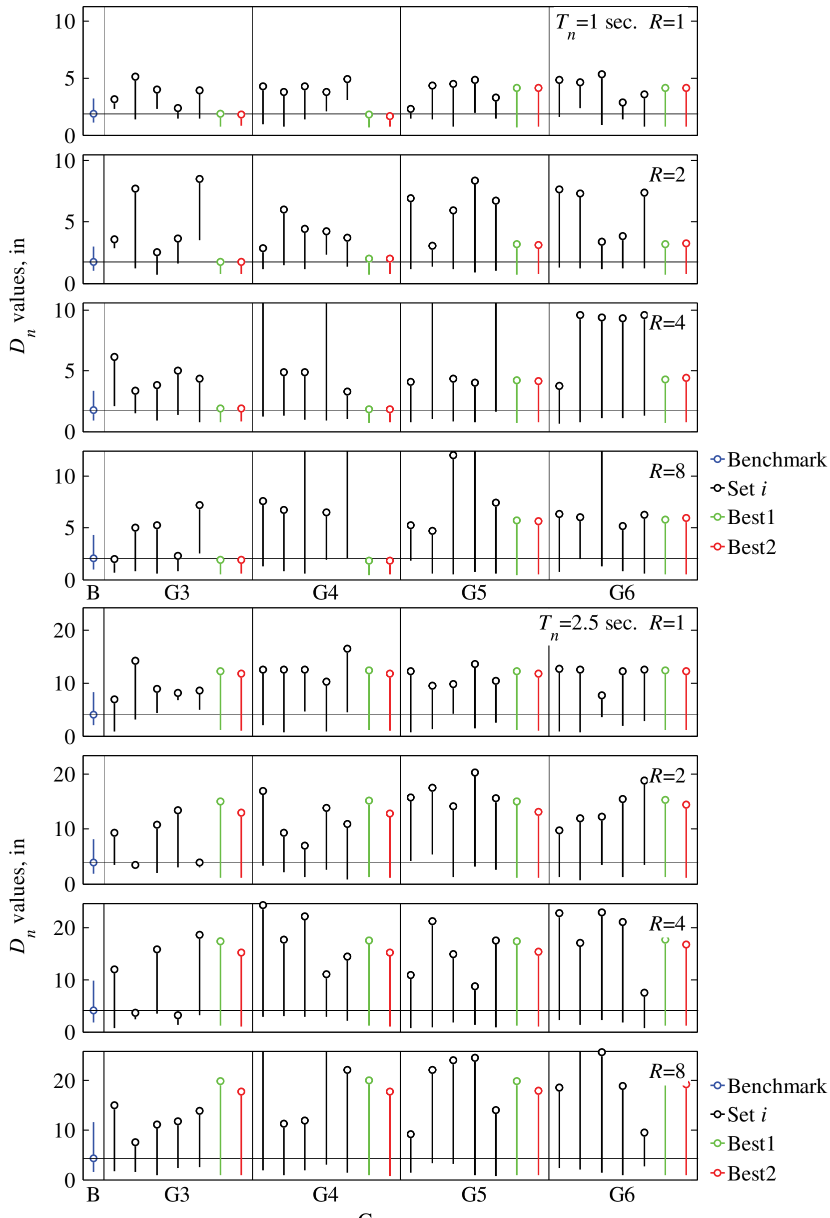

Figure 5. Range of inelastic deformation values for bilinear systems with $T_n = 1$ (top panels) and 2.5 sec (bottom panels), and $R = 1$, 2, 4, and 8 for different sets of 3, 4, 5, and 6 ground motion records denoted respectively as G3, G4, G5, and G6. The blue dot and the vertical line represent the benchmark (B) median deformation value $\pm\sigma$ assuming a lognormal distribution. For each set, the vertical line and the dot represent the range of the data set and maximum deformation value, respectively.

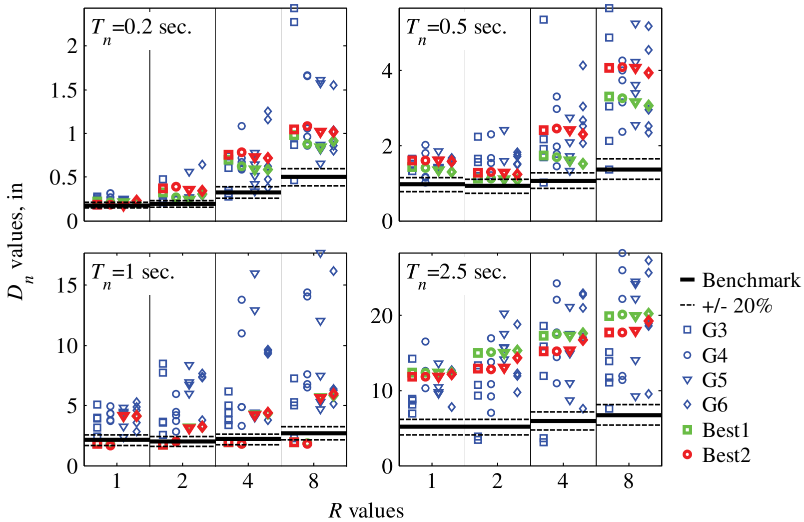

Figure 6. Comparison of benchmark and inelastic deformation values due to ASCE/SEI-7 for bilinear systems. The benchmark EDPs correspond to the mean of 30 deformation values. The deformation values for each set scaled by the ASCE/SEI-7 procedure are obtained as the maximum deformation values of 3, 4, 5, and 6 records in each of seven sets. Included also are sets Best1 and Best2.

750% if R changes from 1 to 8. (3) The improvement gained by the use of sets "Best1" and "Best2" is marginal. For R equal 8, the errors range from 10% to 370% and from 10% to 350% for sets Best1 and Best2, respectively. For elastic–perfectly plastic systems, the errors are larger than those for bilinear systems (Reyes and Kalkan 2011).

The benchmark results shown in Figures 4 and 5 are based on the median deformation value. Figure 6 compares the benchmark, calculated this time as the mean of 30 D_n values and the inelastic deformation values due to ASCE/SEI-7 for bilinear systems. Included also are the horizontal lines at 0.8 and 1.2 times the benchmark to indicate ±20% error around the "true" value. It is apparent that the ASCE/SEI-7 scaling procedure is not accurate and overly conservative as compared to the benchmark results. Insignificant improvement is gained by the use of sets "Best1" and "Best2."

The dispersion measure, δ is used next to evaluate the efficiency and consistency of the ASCE/SEI-7 procedure. δ of n observed values of x_i, are calculated from

$$\delta = \left[\frac{\sum_{i=1}^{n} (\ln x_i - \ln \hat{x})^2}{n-1} \right]^{1/2} \tag{1}$$

where $\hat{x}$ is defined as the median value

$$\hat{x} = \exp\left[\frac{\sum_{i=1}^{n} \ln x_i}{n}\right] \tag{2}$$

Intraset dispersion, which is implied by the vertical lines in Figures 4 and 5, was calculated using Equation 1 taking the deformation estimates within a set as observed values x_i. The scatter of the dots in Figures 4 and 5 was measured by calculating interset dispersion values. Intraset and interset dispersion plots presented respectively in Figures 7 and 8 show that the intraset dispersion increases with increasing period and R value, implying that the procedure becomes less efficient. Similarly, interset dispersion increases with increasing period and R value, indicating that the procedure becomes less consistent. Similar results obtained for elasto-plastic systems were reported in Reyes and Kalkan (2011). According to the results presented in Figures 4 through 8, the accuracy, efficiency and consistency in the estimation of inelastic deformations are not achieved in the ASCE/SEI-7 procedure if fewer than seven records are employed. Therefore the procedure inherently penalizes the analyst for using fewer than seven records in nonlinear RHAs (personal communication with Nico Luco).

EVALUATION OF ASCE/SEI-7 SCALING PROCEDURE: SEVEN OR MORE GROUND MOTIONS

The accuracy of the ASCE/SEI-7 procedure was evaluated next by comparing the median (or mean) value of the inelastic deformation D_n due to seven sets of 7 to 10 scaled records

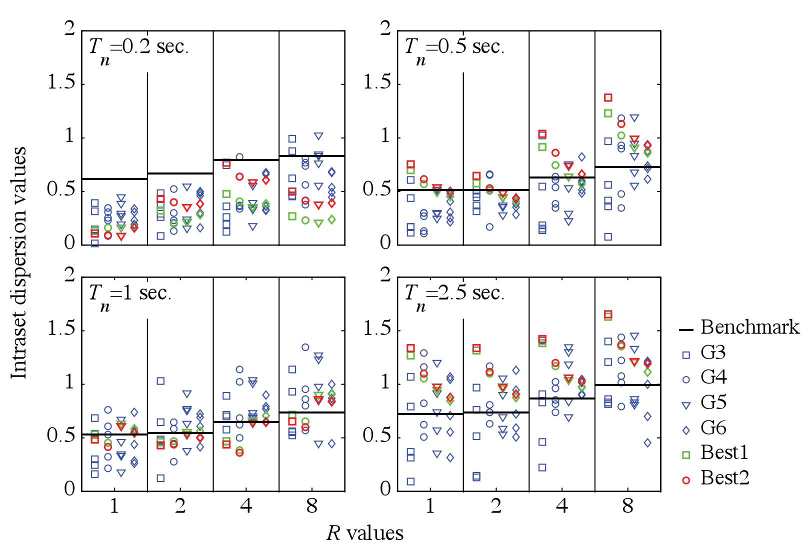

Figure 7. Benchmark and ASCE/SEI-7 intraset dispersion values for bilinear systems. Lognormal distribution is assumed. Larger intraset dispersion indicates larger variability of response values within a set; that implies inefficiency.

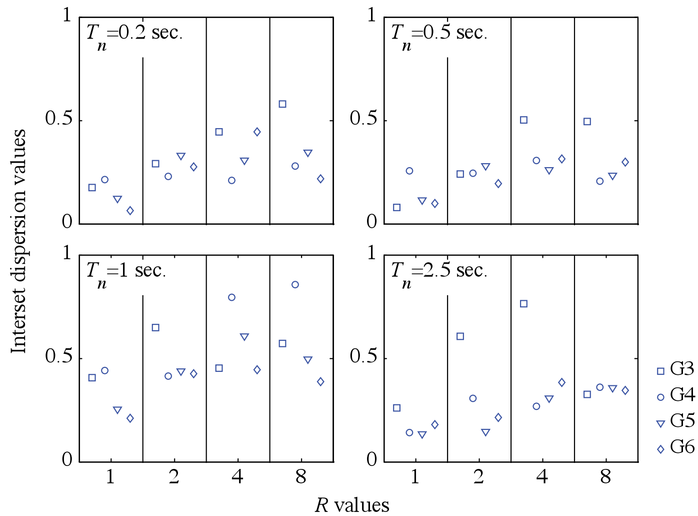

Figure 8. ASCE/SEI-7 interset dispersion values for bilinear systems. Lognormal distribution is assumed. Larger interset dispersion indicates larger set-to-set variability; that implies inconsistency.

against the benchmark value, defined as the median (or mean) value of D_n due to the 30 unscaled ground motions. Figures 9 and 10 show the range of inelastic deformation values D_n for bilinear systems due to groups of 7, 8, 9, and 10 records, called as G7, G8, G9, and G10, respectively. As explained previously, seven sets were considered in each of these groups (that is, a total of 28 sets, plus sets Best1 and Best2 for each T_n, R, and constitutive model combinations). The dot and the vertical line represent the median deformation value $\pm\sigma$ assuming a lognormal distribution. Figures 9 and 10 permit the following observations: (1) Increasing the number of records from 7 to 10 has a minor effect in the accuracy of the procedure; overestimations range from 3% to 155% and from 0.6% to 123% for groups G7 and G10, respectively. (2) The accuracy of the procedure decreases with increasing R value and increasing period T_n; the maximum error increases from 74% to 155% and from 79% to 155% if R changes from 1 to 8, and T_n changes from 0.2 to 2.5 sec, respectively. Figure 11 compares the benchmark results with the inelastic deformation values due to the ASCE/SEI-7 scaling procedure for bilinear systems for the range of R values considered; in this case, mean values are used for both the ASCE/SEI-7 and the benchmark results. Included also in this figure are the horizontal lines at 0.8 and 1.2 times the benchmark to represent ±20% error around the "true" value. By comparing Figure 6 with Figure 11, it is obvious that when seven or more randomly selected records are used, the data points are less scattered and closer to the mean benchmark prediction; therefore, the procedure provides more accurate estimate of inelastic deformations. However, the overestimations in median values of inelastic deformation are generally larger than 20%, especially for $R = 4$ and 8. Similar conclusions are obtained for elastic–perfectly plastic systems (Reyes and Kalkan 2011).

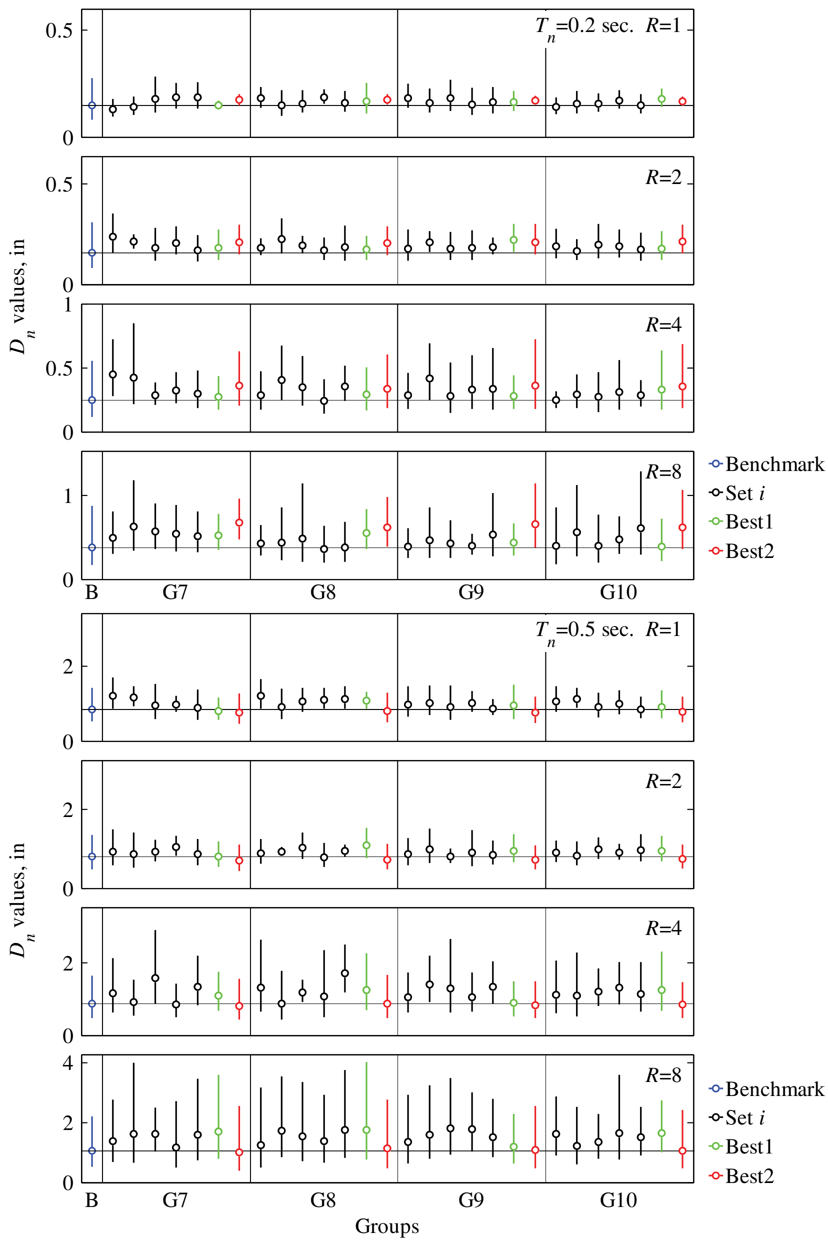

Figure 9. Range of inelastic deformation values for bilinear systems with $T_n = 0.2$ (top panels) and 0.5 sec (bottom panels), and $R = 1$, 2, 4, and 8 for different sets of 7, 8, 9, and 10 ground motion records denoted respectively as G7, G8, G9, and G10. The blue dot and the vertical line represent the benchmark (B) median deformation value $\pm\sigma$ assuming a lognormal distribution. For each set, the vertical line and the dot represent the range of the data set and median deformation value, respectively.

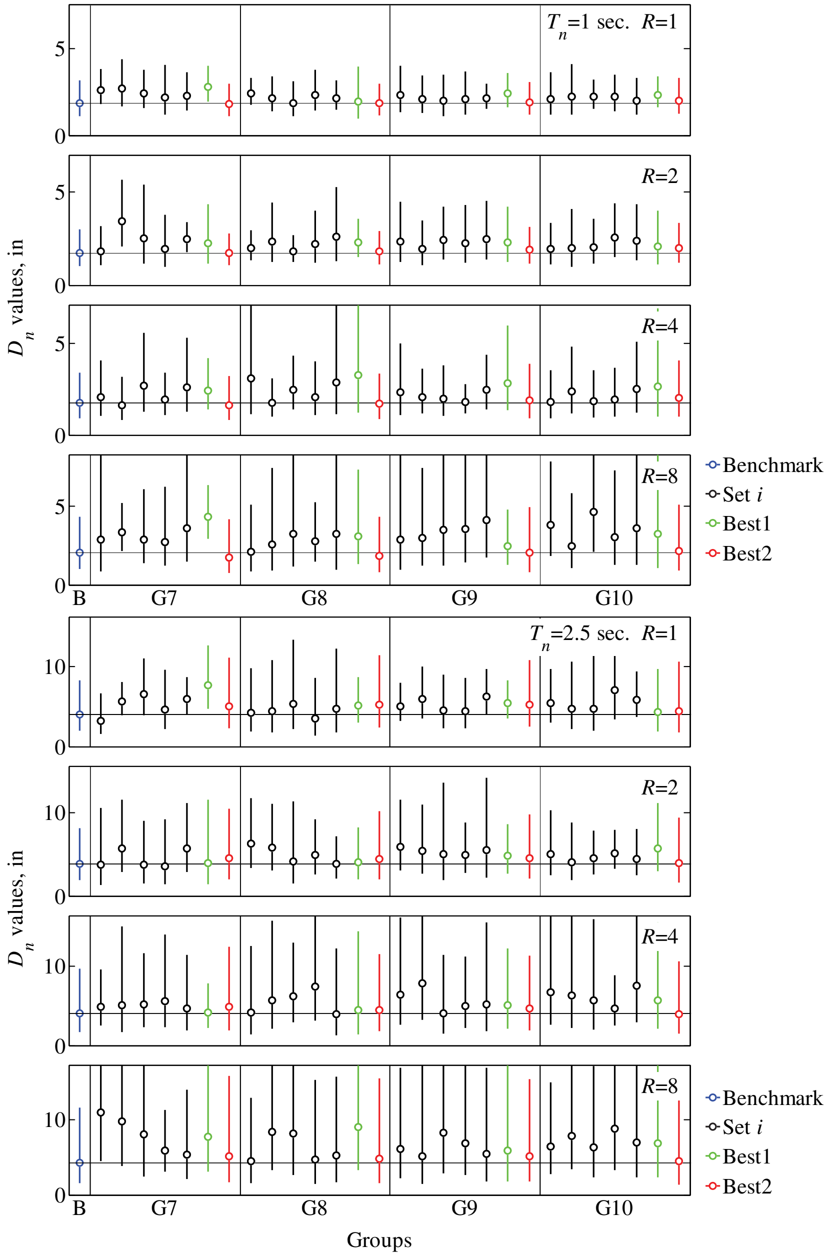

Figure 10. Range of inelastic deformation values for bilinear systems with $T_n = 1$ (top panels) and 2.5 sec (bottom panels), and $R = 1$, 2, 4, and 8 for different sets of 7, 8, 9, and 10 ground motion records denoted respectively as G7, G8, G9, and G10. The blue dot and the vertical line represent the benchmark (B) median deformation value $\pm\sigma$ assuming a lognormal distribution. For each set, the vertical line and the dot represent the range of the data set and median deformation value, respectively.

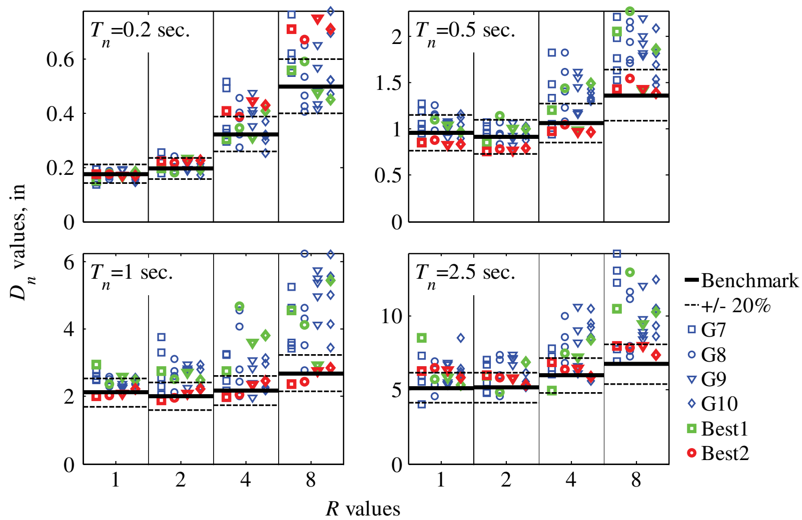

Figure 11. Benchmark and inelastic deformation values due to ASCE/SEI-7 for bilinear systems. The benchmark EDPs correspond to the mean of 30 deformation values. The deformation values for each set scaled by the ASCE/SEI-7 procedure are obtained as the mean of 7, 8, 9, and 10 deformation values. Included also are sets Best1 and Best2.

For systems with short periods and large R values, the mean of randomly selected sets is not similar to the mean of the benchmark data set as demonstrated by the analysis of variance (ANOVA) values in Figure 12. In this figure, data points for G7-G10, $T_n = 0.2$s, 0.5s, and $R = 2, 4, 8$ are hidden by the red and green points. The ANOVA test returns the p value under the null hypothesis that both ASCE/SEI-7 and benchmark results are drawn from populations with the same mean. If p is near zero, it questions the null hypothesis and suggests that the mean of scaled set according to the ASCE/SEI-7 is significantly different than the benchmark mean. This statistical test indicates that the random selection of records for the ASCE/SEI-7 procedure may lead to inconsistent results.

For systems with $T_n > 0.5$ sec or small R values, set "Best2" is much more accurate than set "Best1" demonstrating that consideration of spectral shape and also $A(T_n)$ in selecting and scaling ground motions improves the D_n estimates significantly (Figs. 7 through 9). For $T_n = 2.5$ sec, the error of the procedure ranges from 2% to 109% and from 1% to 28% for sets "Best1" and "Best2," respectively. For systems with very short periods ($T_n = 0.2$ sec) and large R values (4 and 8), both sets "Best 1" and "Best 2" lead to inaccurate estimates of inelastic deformations (Figures 7 through 9); overestimations exceed 40% for bilinear systems and 100% for elastic–perfectly plastic systems (Reyes and Kalkan 2011). This is due to the high variability of spectral pseudo-accelerations and large discrepancies between elastic and inelastic spectra for periods in the acceleration sensitive region and large R values.

The intraset and interset dispersion values are shown next in Figures 13 and 14 for bilinear systems, where a lognormal distribution of D_n values was assumed. These dispersion

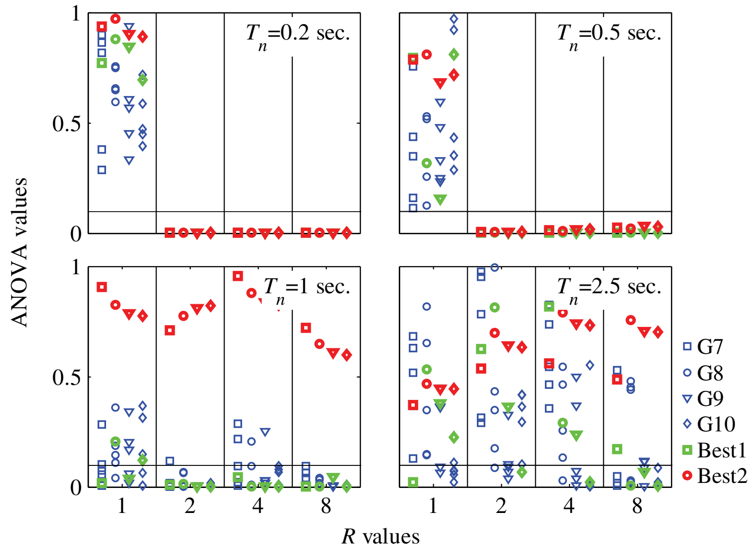

Figure 12. ANOVA values (p) for bilinear systems. The ANOVA test compares deformation values from each set against the benchmark deformation values. Near zero p values suggest that the mean of scaled set according to the ASCE/SEI-7 is significantly different than the benchmark mean.

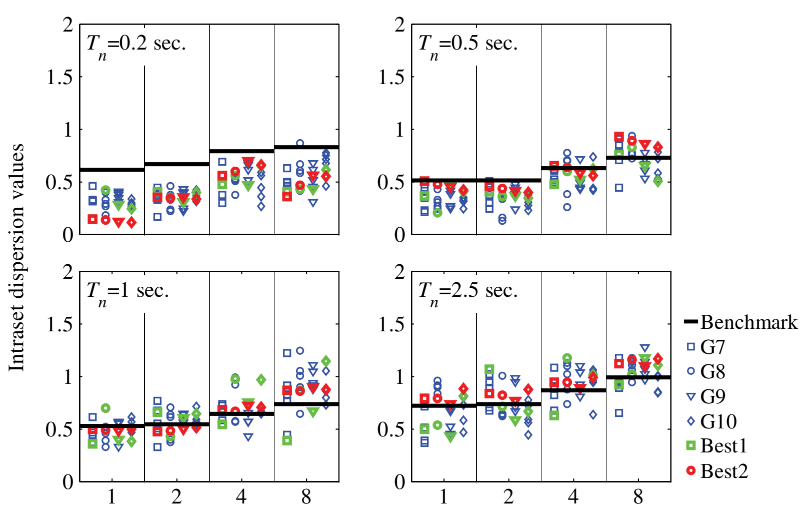

Figure 13. Benchmark and ASCE/SEI-7 intraset dispersion values for bilinear systems. Lognormal distribution is assumed. Larger intraset dispersion indicates larger variability of response values within a set; that implies inefficiency.

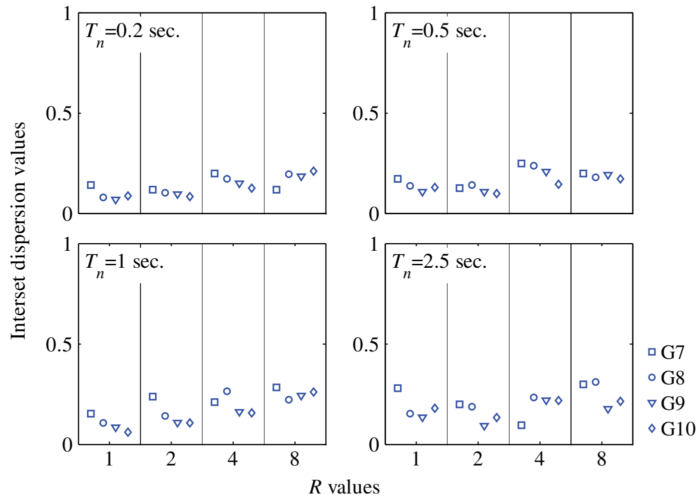

Figure 14. ASCE/SEI-7 interset dispersion values for bilinear systems. Lognormal distribution is assumed. Larger interset dispersion indicates larger set-to-set variability; that implies inconsistency.

quantities were calculated using Equation 1 as explained before. The intraset dispersion increases with increasing R values, indicating larger variability of response values within a set. As expected, the interset dispersion tends to decrease with increasing number of records per set. Results presented in Reyes and Kalkan (2011) evidence that utilizing seven or more randomly selected records in the ASCE/SEI-7 reduces the interset dispersion significantly; this reduction is more pronounced for elastic–perfectly plastic systems. The reduced interset variability indicates the consistency in the benchmark estimates of the ASCE/SEI-7 procedure using different sets of records. For systems with a fundamental period in the velocity or displacement sensitive region, accuracy, efficiency and consistency are achieved only if records are selected on the basis of their spectral shape and $A(T_n)$ as opposed to random selection.

MODAL-PUSHOVER-BASED SCALING PROCEDURE: ALTERNATIVE TO THE ASCE/SEI-7 SCALING PROCEDURE

Because the ASCE/SEI-7 ground motion scaling method does not consider explicitly the inelastic behavior of the structure (that is, strength), it may not be appropriate for structures with short periods or for structures located in near-field sites where the inelastic deformation can be significantly larger than the deformation of the corresponding linear system. For such cases, scaling methods that are based on the inelastic deformation spectrum or methods that consider the response of the first-"mode" inelastic SDOF system are more appropriate (Luco and Cornell, 2007; Tothong and Cornell, 2008; PEER, 2009). Kalkan and Chopra (2010, 2011, 2012) used these concepts to develop a modal pushover-based scaling (MPS) procedure for selecting and scaling earthquake ground motion records in a form convenient for evaluating existing structures and proposed designs of new building structures. This procedure tested subsequently for bridges in Kalkan and Kwong (2011, 2012). The MPS procedure explicitly considers structural strength, determined from the first-"mode" pushover curve, and determines a scaling factor for each record to match a target value of the deformation of the first-"mode" inelastic SDOF system. If the MPS procedure were applied to the systems of this investigation, it would lead to practically null error in the estimation of inelastic deformations and null intraset and interset dispersions. Therefore, the MPS procedure for SDOF systems would be accurate, efficient, and consistent.

CONCLUSIONS

Based on elastic–perfectly plastic and bilinear inelastic single-degree-of-freedom systems, the accuracy, efficiency, and consistency of the ASCE/SEI-7 ground motion scaling procedure are examined by comparing the median and mean values of the inelastic deformation due to 480 sets of scaled records against benchmark results. The number of records in these sets varies from three to ten. The records in each set were selected either (i) randomly, (ii) considering their spectral shapes or (iii) considering the design spectral acceleration value $A(T_n)$ in addition to their spectral shapes. This evaluation of the ASCE/SEI procedure has led to the following conclusions:

1. The ASCE/SEI-7 scaling procedure does not insure a unique scaling factor for each record; obviously, various combinations of scaling factors can be defined to insure that the mean spectrum of scaled records remains above the target spectrum over the specified period range. Utilizing a minimum scale factor closer to unity for each record may overcome this problem.
2. The ASCE/SEI-7 procedure is found to be conservative as compared to the benchmark responses from hazard compatible unscaled records using a large catalog of ground motions. It is neither efficient nor consistent if fewer than seven ground motions are utilized, thus penalizing the analyst for employing fewer than seven ground motions for nonlinear RHAs.
3. The ASCE/SEI-7 scaling procedure utilizing seven or more randomly selected records provides more accurate estimate of inelastic deformations. However, the overestimations in median values of inelastic deformation are generally larger than 20%. Increasing the number of records from 7 to 10 has a minor effect in the accuracy of the procedure. Thus, use of 7 records is found to be sufficient.
4. In general, the accuracy of the procedure decreases with increasing R value. The fundamental period T_n (that is, short period or long period systems) does not affect significantly its accuracy if the records are selected randomly.
5. For systems with $T_n \geq 0.5$ sec or small R values (R<4), consideration of spectral shape and also $A(T_n)$ in selecting and scaling ground motions improves the D_n estimates significantly. For bilinear systems with $T_n = 2.5$ sec, the maximum error of the procedure decreases from 109% to 28% when $A(T_n)$ is considered in addition to spectral shape. For systems with very short periods (0.2 sec) and large R values (4 and 8), however, both sets "Best 1" and "Best 2" lead to inaccurate estimates of inelastic deformations with overestimations exceeding 100% for elastic–perfectly plastic systems and 40% for bilinear systems. This is due to the high variability

of spectral pseudo-accelerations and the large discrepancies between elastic and inelastic spectra for periods in the acceleration sensitive region and large R values (Chopra and Chintanapakdee, 2004). For such cases, scaling methods that are based on the inelastic deformation spectrum or that consider the response of the first-"mode" inelastic SDOF system are more appropriate.

This study has focused on the statistical examination of the required number of records for the ASCE/SEI ground motion scaling method, which has been limited to stable elastic–perfectly plastic and bilinear inelastic single-degree-of-freedom systems.

NOTATION

The following symbols are used in this paper:

$A(T)$	Pseudo-spectral acceleration at period T
$\mathbf{A}$	Vector of pseudo-spectral acceleration values
$\hat{A}(T)$	Target value of pseudo-spectral acceleration at period T
$\hat{\mathbf{A}}$	Vector of target pseudo-spectral acceleration values
$\hat{\mathbf{A}}_{\text{scaled}}$	Mean scaled spectrum of m records
D_n	Peak inelastic deformation value of SDOF system
$\varepsilon_{\text{ASCE}}$	Maximum normalized difference between target and mean scaled spectrum
m	Number of ground motion records
M_w	Moment magnitude
R	Yield-strength reduction factor
R_{JB}	Joyner-Boore distance–perpendicular distance to surface projection of fault plane
SF	Ground motion scaling factor
T_n	Period of single-degree-of-freedom system; or elastic first-"mode" vibration period of structure
V_{S30}	Average shear-wave velocity within 30m depth from surface
σ	Standard deviation

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