Relevance of Fault-Normal/Parallel and Maximum Direction Rotated Ground Motions on Nonlinear Behavior of Multi-Story Buildings

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SUMMARY:
In United States, the seismic provisions require at least two horizontal ground motion components for three-dimensional (3D) response history analysis (RHA) of structures. For sites within 5 km of an active fault, these records should be rotated to fault-normal/fault-parallel (FN/FP) directions, and two RHA analyses should be performed separately (when FN and then FP are aligned with transverse direction of the structural axes). It is assumed that this approach will lead to two sets of responses that envelope the range of possible responses over all non-redundant rotation angles. This assumption is examined here using 3D computer models of 9-story structures having symmetric (torsional-stiff) and asymmetric (torsional-flexible) layouts subjected to an ensemble of bi-directional near-fault strong ground motions with and without distinct velocity pulses. The influence that the rotation angle of the ground motion has on several engineering demand parameters (EDPs) is examined in nonlinear-inelastic domain to form a benchmark for evaluating the use of the FN/FP directions. In a similar way, we have also examined the maximum-direction (MD) ground motion, a revised definition of horizontal ground motions for use in site-specific ground motion procedures for seismic design.

Keywords: Fault-Normal/Parallel Direction; Maximum-Direction Ground Motion; Response History Analysis

1. INTRODUCTION

In United States, both the California Building Code (ICBO, 2010) and International Building Code (ICBO, 2009) refer to ASCE/SEI 7-05 Chapter 16 (ASCE, 2005) when response history analysis (RHA) is required for design verification of building structures. According to the ASCE/SEI 7-05 provisions, at least two horizontal ground motion components should be considered for three-dimensional (3D) RHA of structures. The California Building Code (ICBO, 2010) also requires that at sites within 5 km of the active fault that dominates the hazard, each pair of ground motion components should be rotated to the fault-normal and fault-parallel (FN/FP) directions. It is assumed that this approach will lead to two sets of responses that envelope the range of possible responses over all non-redundant rotation angles. This modification, which was absent in ASCE/SEI 7-05, is now included in ASCE/SEI 7-10 (ASCE, 2010), which has additional proposed changes to be incorporated in the new generation of the building codes. One of these changes is the use of maximum-direction (MD) ground motion, a revised definition of horizontal ground motions for use in site-specific ground motion procedures for seismic design.

Using 3D computer models of two 9-story buildings having symmetric (torsional-stiff) and asymmetric (torsional-flexible) lay outs, and an ensemble of bi-directional near-fault strong ground motions with and without distinct velocity pulses, this study examines the influence that the rotation angle (on horizontal plane) of the ground motion has on several engineering demand parameters (EDPs) to form a benchmark for evaluating the use of the FN/FP directions as well as maximum direction for nonlinear RHA. Also investigated are the rotation angle of an apparent velocity-pulse and its correlation with the FN/FP direction, as well as with the rotation angle of ground motion corresponding to peak structural response quantities. At the end, this study provides recommendations towards the use of ground motions rotated to FN/FP directions and maximum direction.

2. GROUND MOTIONS SELECTED

The thirty near-fault strong motion records (Table 1) selected for this investigation were recorded from nine shallow crustal earthquakes compatible with the following scenario: Moment magnitude = 6.7±0.2; Closest distance < 15 km; Record highest usable period ≥ 6 s.

Table 1. Selected near-fault strong ground motion records

Record sequence number	Earthquake name	Year	Station name	Earthquake magnitude (M_w)	Style of Faulting	Closest fault distance (km)
1	Gazli, USSR	1976	Karakyr	6.8	Thrust	5.5
2	Imperial Valley-06	1979	Aeropuerto Mexicali	6.5	Strike-slip	0.3
3	Imperial Valley-06	1979	Agrarias	6.5	Strike-slip	0.7
4	Imperial Valley-06	1979	Bonds Corner	6.5	Strike-slip	2.7
5	Imperial Valley-06	1979	EC Meloland Overpass FF	6.5	Strike-slip	0.1
6	Imperial Valley-06	1979	El Centro Array #6	6.5	Strike-slip	1.4
7	Imperial Valley-06	1979	El Centro Array #7	6.5	Strike-slip	0.6
8	Irpinia, Italy-01	180	Auletta	6.9	Normal	9.6
9	Irpinia, Italy-01	1980	Bagnoli Irpinio	6.9	Normal	8.2
10	Irpinia, Italy-01	1980	Sturno	6.9	Normal	10.8
11	Nahanni, Canada	1985	Site 1	6.8	Thrust	9.6
12	Nahanni, Canada	1985	Site 2	6.8	Thrust	4.9
13	Nahanni, Canada	1985	Site 3	6.8	Thrust	5.3
14	Superstition Hills-02	1987	Parachute Test Site	6.5	Strike-slip	1.0
15	Superstition Hills-02	1987	Westmorland Fire Sta	6.5	Strike-slip	13.0
16	Loma Prieta	1989	BRAN	6.9	Reverse	10.7
17	Loma Prieta	1989	Gilroy Array #3	6.9	Reverse	12.8
18	Loma Prieta	1989	LGPC	6.9	Reverse	3.9
19	Loma Prieta	1989	San Jose - Santa Teresa Hills	6.9	Reverse	14.7
20	Loma Prieta	1989	Saratoga - Aloha Ave	6.9	Reverse	8.5
21	Loma Prieta	1989	Saratoga - W Valley Coll.	6.9	Reverse	9.3
22	Erzican, Turkey	1992	Erzincan	6.7	Strike-slip	4.4
23	Northridge-01	1994	Jensen Filter Plant Generator	6.7	Reverse	5.4
24	Northridge-01	1994	Newhall - Fire Sta	6.7	Reverse	5.9
25	Northridge-01	1994	Newhall - W Pico Canyon Rd.	6.7	Reverse	5.5
26	Northridge-01	1994	Pacoima Dam (downstr)	6.7	Reverse	7.0
27	Northridge-01	1994	Rinaldi Receiving Sta	6.7	Reverse	6.5
28	Northridge-01	1994	Sylmar - Olive View Med FF	6.7	Reverse	5.3
29	Kobe, Japan	1995	KJMA	6.9	Reverse	1.0
30	Kobe, Japan	1995	Nishi-Akashi	6.9	Reverse	7.1

These ground motions were rotated to the fault-normal (FN) and fault-parallel (FP) orientations using the following planer transformation equations:

$$\ddot{u}_{FP} = \ddot{u}_1 \cos(\beta_1) + \ddot{u}_2 \cos(\beta_2) \tag{1}$$

$$\ddot{u}_{FN} = \ddot{u}_1 \sin(\beta_1) + \ddot{u}_2 \sin(\beta_2) \tag{2}$$

where $\beta_1 = \alpha_{strike} - \alpha_1$, $\beta_2 = \alpha_{strike} - \alpha_2$, α_{strike} is the strike of the fault, α_1 and α_2 are the azimuths of the instrument axes as shown in Figure 1a. Shown in Figure 2 are the 5-percent-damped geometric-mean (termed “median” here after) response spectra for the FN and FP components of the unscaled ground motions. As expected, the ordinates of median spectra of FN components are larger than those of FP components, because the FN components of near-fault ground motions are generally stronger.

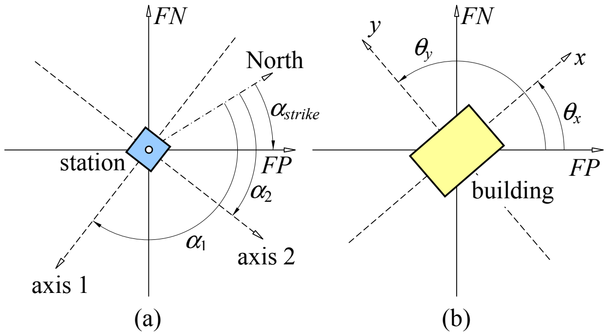

Figure 1. (a) Reference axes for the fault strike and the instrument with relevant angles noted. (b) Reference axes for the building (FN = Fault-normal; FP = Fault-parallel)

The ground motions were additionally rotated θ_x° away from the FP axis as shown in Figure 1b. The angle θ_x varies from 10° to 360° every 10°. These rotations were conducted using equations (1) and (2) with the following modifications: (a) α_1 and α_2 were changed by θ_x and θ_y, respectively; (b) β_1 and β_2 were redefined as $\beta_1=\alpha_{strike}-\alpha_1-\theta_x$ and $\beta_2=\alpha_{strike}-\alpha_2-\theta_y$. The x and y axes as well as the angles θ_x and θ_y are shown in Figure 1b.

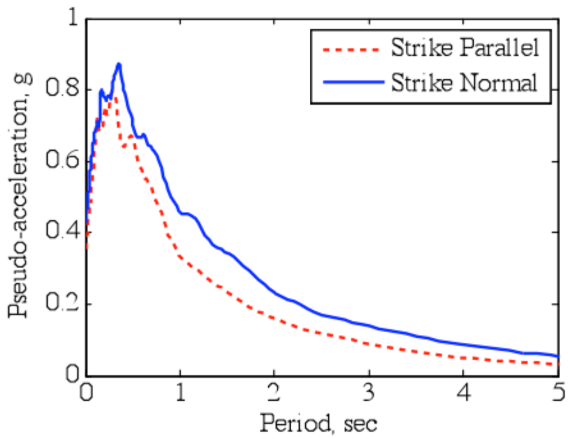

Figure 2. Geometric-mean of thirty response spectra of the selected near-fault ground-motion components in fault-normal and fault-parallel directions. Damping ratio 5-percent

Figure 3 shows the response of a two-degree-of-freedom system with equal stiffness and damping ratio in the x and y directions subjected to the FN/FP components of a ground motion (i.e. $\theta_x = 0$). The maximum deformation of this system occurs at an angle θ_m away from the FP axis. This new orientation for the response quantity of interest is called as maximum-direction (MD). In the literature, the two perpendicular axes rotated θ_m from the FP axis are commonly called major (MA) and minor (MI) axes or simply principal axes.

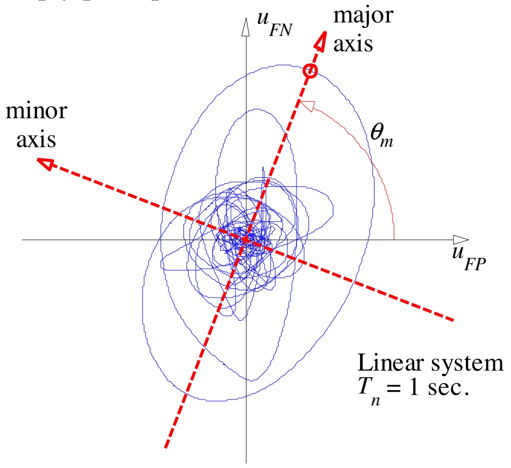

Figure 3. Trace of deformation orbit of a two-degree-of-freedom system with direction-independent stiffness and damping subjected to the fault-normal/parallel (FN/FP) components of a ground motion; the maximum deformation occurs at θ_m

The maximum-direction (MD) ground motions operate under the assumption that the dynamic properties of the structure (e.g., stiffness, strength) are identical in all directions. This assumption may be true for some in-plan symmetric structures, however, the response of most structures is dominated by modes of vibration along specific axes (e.g., longitudinal and transverse axes in a building), and often the dynamic properties (especially stiffness) along those axes are distinct. It is argued that in order to achieve structural designs consistent with the collapse risk level given in the ASCE/SEI 7-10 documents, the design spectra should be compatible with expected levels of ground motion along those principal response axes (Stewart et al., 2011). The use of MD ground motions effectively assumes that the azimuth of maximum ground motion coincides with the directions of principal structural response. Because this case is unlikely, design ground motions have lower probability of occurrence than intended (Singh et al., 2011).

3. VELOCITY PULSES

In the proximity of an active fault system, ground motions are significantly affected by the faulting mechanism, direction of rupture propagation relative to the site (e.g., forward directivity), as well as the possible static deformation of the ground surface associated with fling-step effects (Kalkan and Kunnath, 2006). These near-source effects cause most of the seismic energy from the rupture to arrive in a single coherent long-period pulse of motion (Kalkan and Kunnath, 2007, 2008). Ground motions having such a distinct pulse-like character arise in general at the beginning of the seismogram, and their effects tend to increase the pseudo-acceleration in the long-period portion of the spectrum (Golesorkhi and Gouchon, 2002). Baker (2007) developed a numerical procedure to identify and characterize such velocity pulses for ground motion records. We use his procedure to identify velocity pulses in rotated motions whose rotation angle is varied from 10° to 360° at an interval of 10°. Results of these analyses are presented in Reyes and Kalkan (2012a). Identification of velocity pulses helps us to understand their importance when ground motions are rotated to FN/FP and maximum directions.

4. STRUCTURAL SYSTEMS AND COMPUTER MODELS

The symmetric-plan structure considered is an existing 9-story steel building with ductile frames (Fig. 4) designed as an office building according to 2001 California Building Code. The lateral load resisting system consists of two ductile steel moment frames in the longitudinal and transverse directions. The asymmetric building selected (Fig. 4) is a hypothetical steel building with ductile frames designed according to the 1985 Uniform Building Code (UBC85). Both buildings are modeled for dynamic analysis, implemented by the PERFORM-3D computer program (CSI, 2006). The 3D model of the symmetric building has the following features: (1) Beams and columns are modelled by a linear element with tri-linear plastic hinges at the ends of the elements that can include in-cycle strength deterioration, but not cyclic stiffness degradation. The bending stiffness of the beams is modified to include the effect of the slab. Axial load-moment interaction in columns is based on plasticity theory. (2) Panel zones are modelled as four rigid links hinged at the corners with a rotational spring that represents the strength and stiffness of the connection (Krawinkler, 1978). (3) The tab connections are modelled using rigid-perfectly-plastic hinges that can include in-cycle and cyclic degradation. (4) The contribution of non-structural elements is modelled by adding four shear columns located close to the perimeter of the building with their properties obtained from simplified models of the façade and partitions. Nonlinear behaviour of these elements is represented using rigid-plastic shear hinges. (5) Ductility capacities of girders, columns and panel zones are specified according to the ASCE/SEI 41-06 standard (ASCE, 2007). (6) Columns of moment resisting frames and the gravity columns are assumed to be clamped at the base. (7) A standard P-Delta formulation is used to approximate effects of nonlinear geometry at large deformations for both moment and gravity frames.

The asymmetric building's model has the following features: (1) Beams and columns were modeled by a linear element with tri-linear plastic hinges at the ends of the elements that include in-cycle strength deterioration, but not cyclic stiffness degradation; axial load-moment interaction in columns is represented by plasticity theory. (2) Panel zones were modeled as four rigid links hinged at the corners with a rotational spring that represents the strength and stiffness of the connection. (3) Ductility capacities of girders, columns and panel zones were specified according to the ASCE/SEI 41-06 standard. (4) Columns of moment resisting frames were assumed to be fixed at the base, whereas gravity columns were considered pinned at the base. (5) The geometric nonlinear effects were considered by a standard P-Delta formulation for both moment and gravity frames. (6) Accidental torsion was not considered in the design of the UBC85 building. For both buildings, plots of mode shapes, effective modal masses are given in Reyes and Kalkan (2012a).

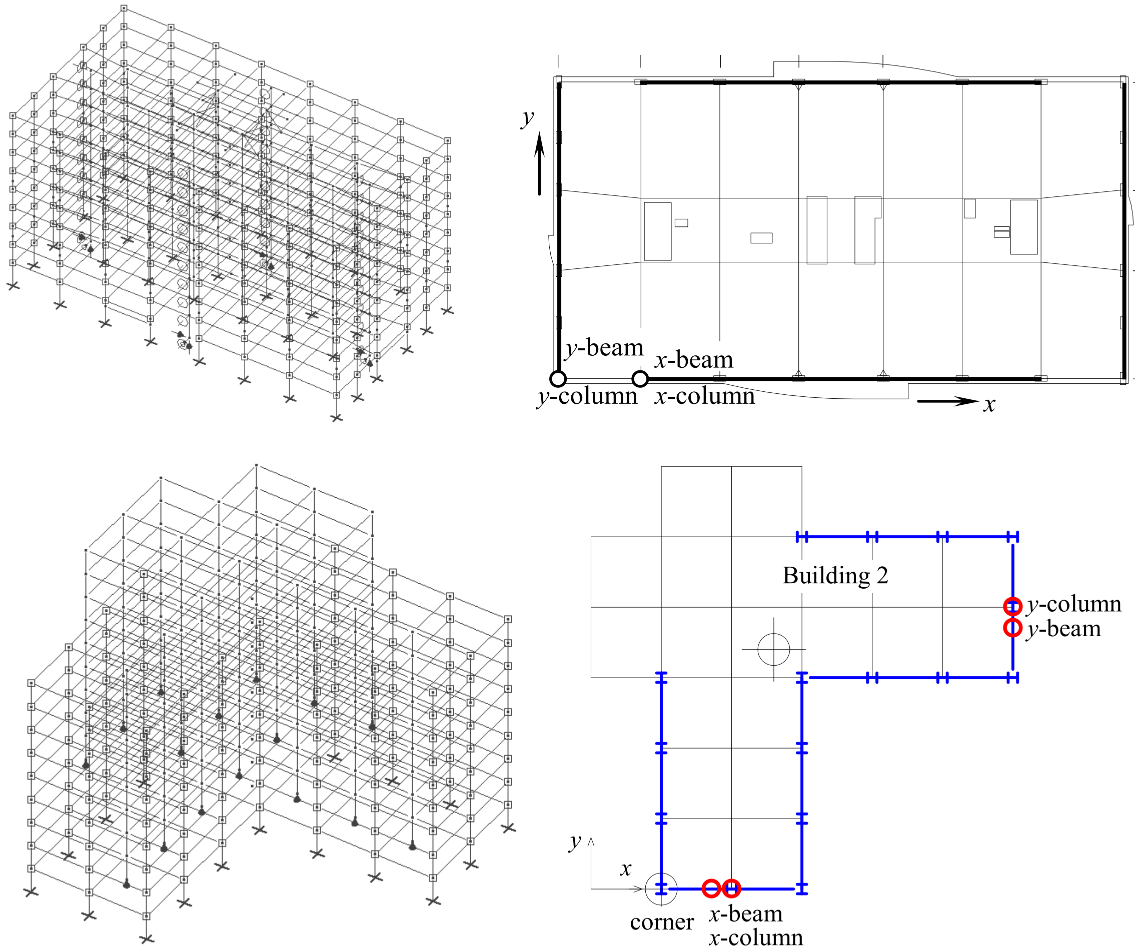

Figure 4. (Top) Nine-story symmetric-plan building; (Bottom) nine-story asymmetric-plan building

5. EVALUATION METHODOLOGY

The following steps were implemented for evaluating the significance of the ground motion rotation angle on nonlinear behavior of buildings in near fault sites:

1. For each of the thirty records selected for this investigation, calculate rotated ground motion components by varying θ_x from 0° to 360° every 10° (Fig. 2b). The motions for θ_x=0° and 90° correspond to the FP and FN components of the record, respectively. In addition, calculate rotated ground motion components for $\theta_x=\theta_m$ and $\theta_x=\theta_m+90°$. For estimating θ_m, use fundamental periods of the buildings.
2. Conduct nonlinear RHAs of the buildings subjected to bi-directional rotated components of ground motion obtained in Step 1. For each RHA, obtain floor displacements, total chord rotations, beam and column moments.

6. RESULTS

Nonlinear RHA was implemented for the systems of this investigation subjected to two horizontal components of ground motion following the procedure of Section 5. Figure 5 shows story drifts, total chord rotations, and beam and column moments at the 1st, 3rd, 5th, 7th, and 9th floors as a function of the rotation angle θ_x for the 9-story symmetric-plan building (T_1=1.51 s) subjected to ground motion No. 9 in Table 1, which has a maximum velocity-pulse period of 1.9 s. The filled gray area shows values of θ_x in which the velocity pulses are identified. Angles θ_x=0° and 90° correspond to the fault-parallel and fault-normal axes, respectively. Same response quantities are plotted in Figure 6 for the 9-story asymmetric building (T_1=2.5 s) subjected to ground motion No. 2, which has a maximum velocity-pulse period of 2.4 s. More comprehensive results including other response quantities (that is EDPs) are presented in Reyes and Kalkan (2012a).

Figures 5 and 6 permit the following observations: (1) Velocity-pulses may appear in directions different than the FN-direction; (2) The maximum drift over all non-redundant orientations seems to be polarized in the direction in which apparent velocity-pulse with period close to T_1 is observed; this polarization is almost perfect for symmetric-plan building; (3) Displacements in the *x*-direction may be underestimated by about 10% to 20% if a building is subjected to only the FN/FP components of a pulse-like ground motion; (4) There is no optimum orientation for a given structure maximizing all EDPs; (5) Maximum value of EDP can happen in any direction different than the direction of the velocity pulse.

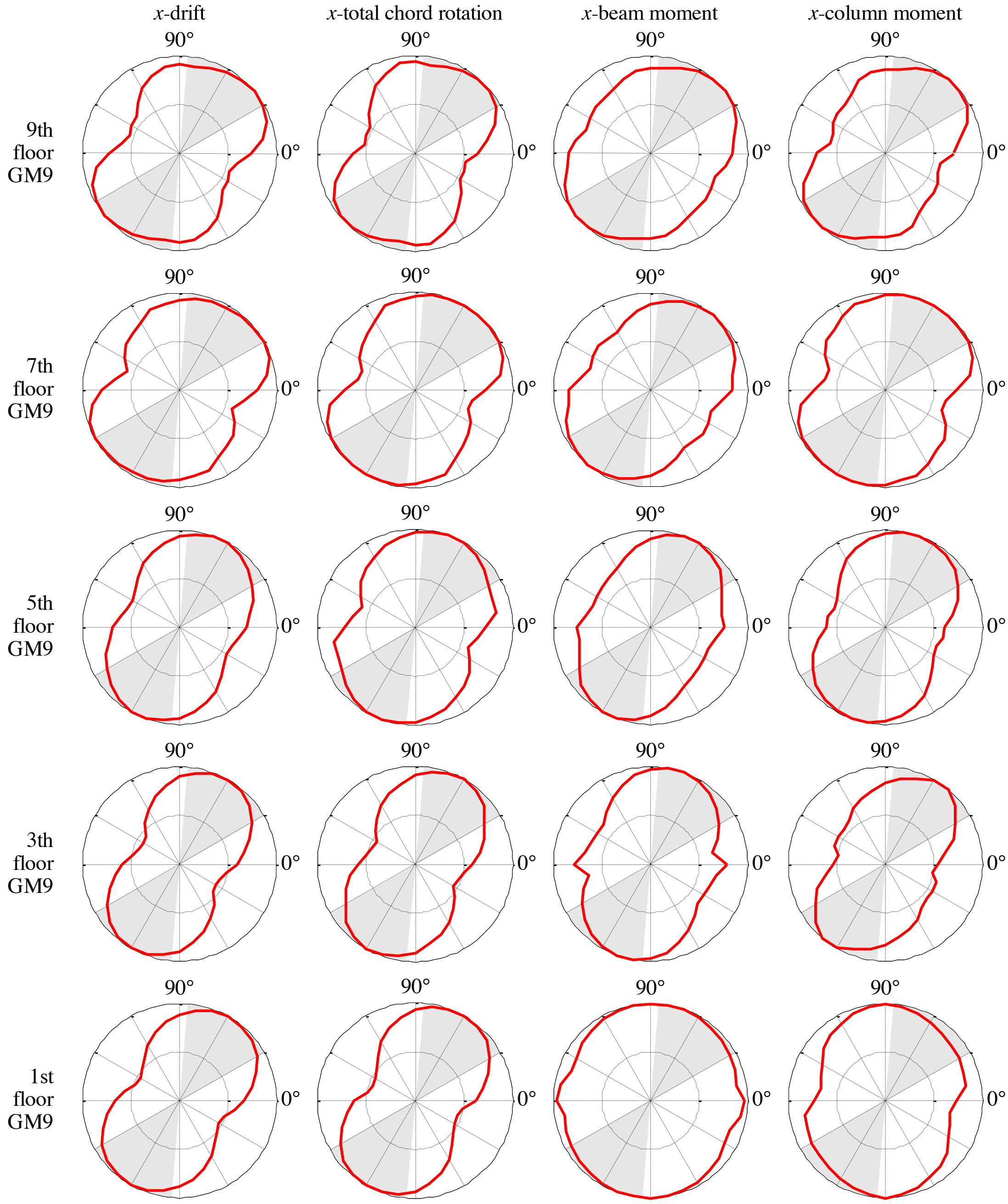

Figure 5. Story drifts, total chord rotations, and internal forces computed at the 1st, 3rd, 5th, 7th, and 9th floors as a function of the rotation angle θx for the nonlinear 9-story symmetric-plan building (T_1=1.51 s) subjected to ground motion No. 9, which has a maximum velocity-pulse period of 1.9 s. The filled gray area shows values of θ_x in which velocity pulses are identified. Angles θ_x=0° and 90° correspond to the fault-parallel and fault-normal axes, respectively [GM = Ground Motion]

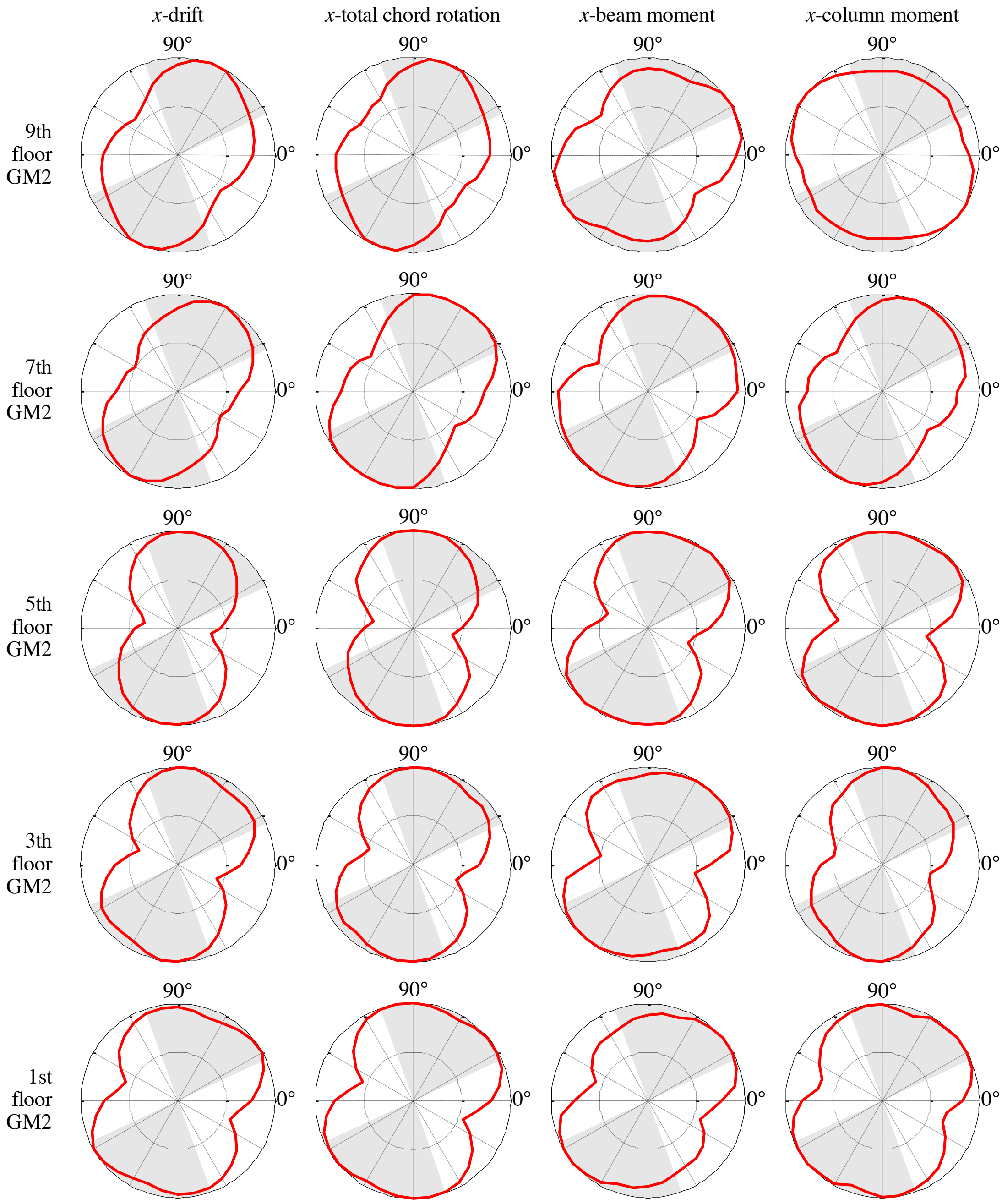

Figure 6. Story drifts, total chord rotations, and internal forces computed at the 1st, 3rd, 5th, 7th, and 9th floors as a function of the rotation angle θ_x for the nonlinear 9-story asymmetric-plan building (T_1=2.5 s) subjected to ground motion No. 2, which has a maximum velocity-pulse period of 2.4 s. The filled gray area shows values of θ_x in which velocity pulses are identified. Angles θ_x=0° and 90° correspond to the fault-parallel and fault-normal axes, respectively [GM = Ground Motion]

For a selected earthquake scenario, it is commonly assumed that EDPs are lognormally distributed (Cornell et al., 2002); for this reason, it is more appropriate to represent the "mean" structural response by the median; a conclusion that is widely accepted. Because the geometric mean and median of a random variable having a lognormal distribution are the same, we decided to employ the term "median" instead of geometric mean, as is commonly done. Figures 7 and 8 show median values of selected EDPs along the x- and y-direction at the 1st, 3rd, 5th, 7th, and 9th floors as a function of the rotation angle θ_x for the asymmetric-plan building subjected to thirty bi-directional ground motions. Same plots for the symmetric building is given in Reyes and Kalkan (2012). The red lines in these figures represent the median ± one standard deviation σ computed based on peak response values due

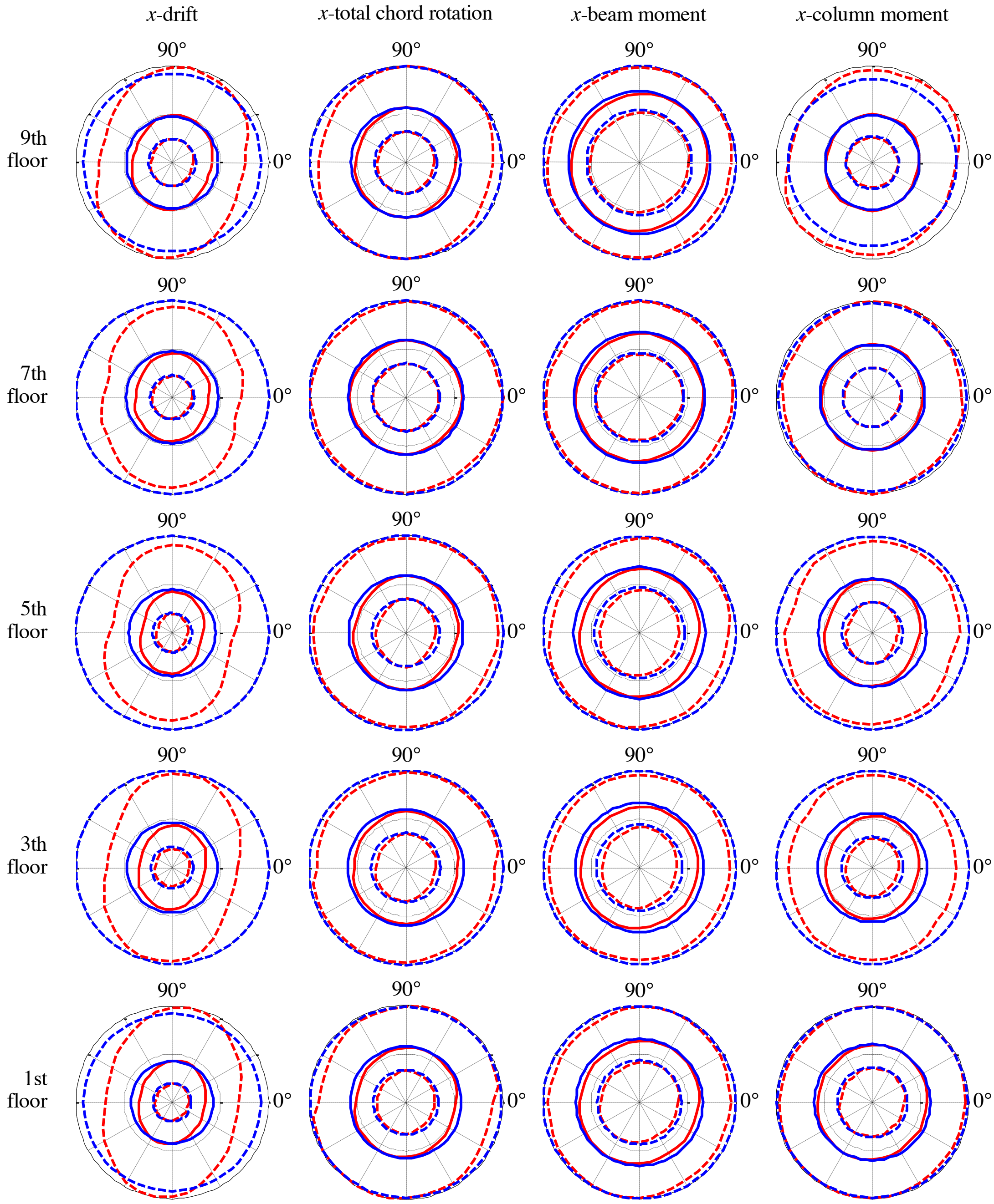

Figure 7. Median values of story drifts, total chord rotations, and internal forces computed at the 1st, 3rd, 5th, 7th, and 9th floors in the x-direction as a function of rotation angle θ_x for the 9-story asymmetric-plan building subjected to bi-directional loading. The red lines represent the median EDP values ± one standard deviation. The blue circles represent the median EDP values ± one standard deviation for the building subjected to bi-directional ground motions in the principal axes.

to each ground motion pair at each non-redundant rotation angle. The blue circles represent the median values EDP at MD (e.g., $drift_{my} \pm \sigma$) for the systems subjected to ground motions only in the principal axes. Recall that MD stands for maximum direction (that is, the specific directions of rotated ground motion pair resulting in peak linear-elastic response quantity of a single lumped mass oscillator as shown in Fig. 3). Note that for a given ground motion pair, principal axes (maximum direction) changes with period. In Figure 7, although the median MD values for each EDP and their ± σ values correspond to a single value for each system, it is visualized as a full circle to facilitate direct comparisons with median EDP values, which is a function of the rotation angle.

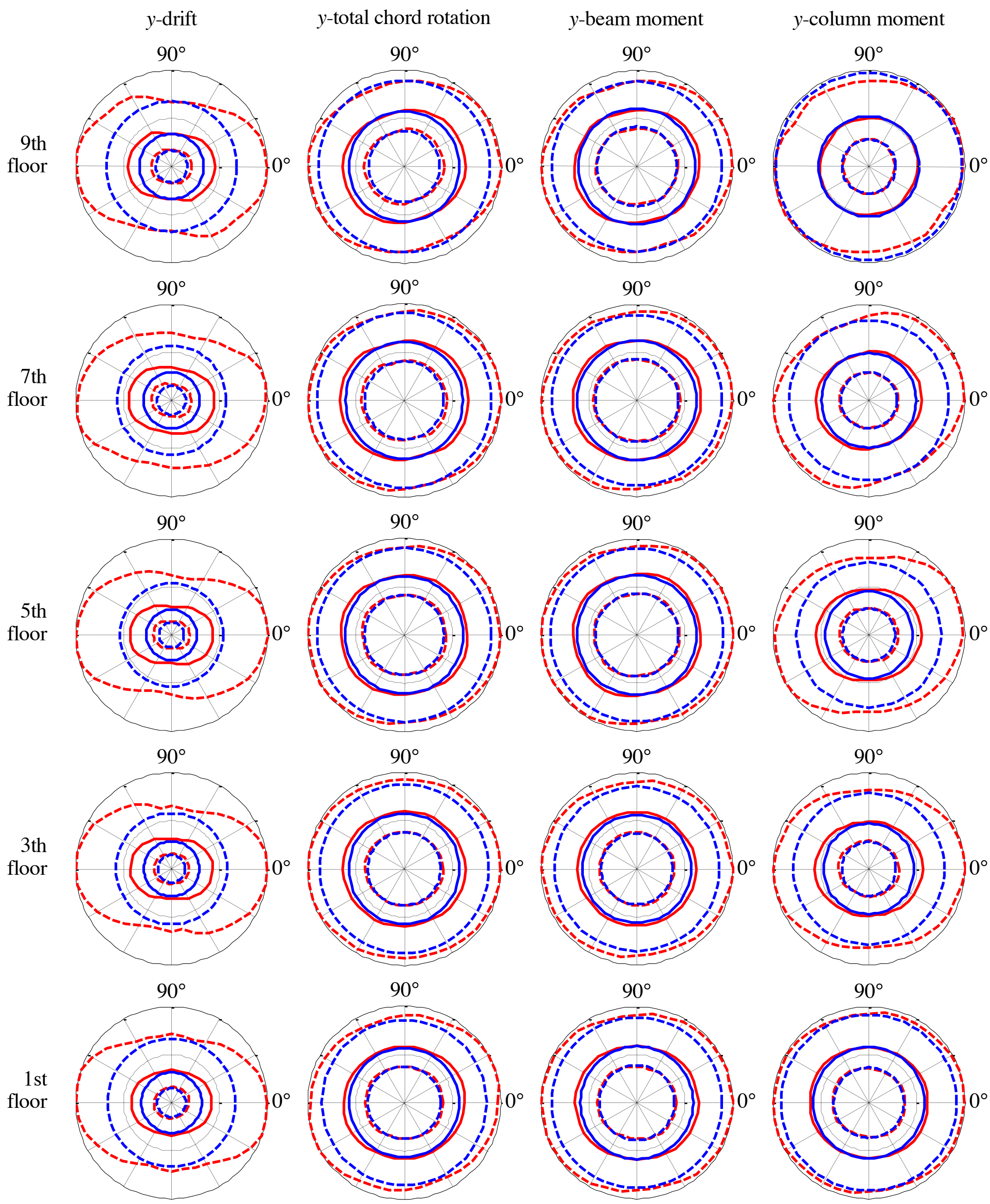

Figure 8. Median values of story drifts, total chord rotations, and internal forces computed at the 1st, 3rd, 5th, 7th, and 9th floors in the y-direction as a function of rotation angle θ_x for the 9-story asymmetric-plan building subjected to bi-directional loading. The red lines represent the median EDP values ± one standard deviation. The blue circles represent the median EDP values ± one standard deviation for the building subjected to bi-directional ground motions in the principal axes.

Figures 7 and 8 demonstrate important information. As such, the maximum median-drift (solid red line) are not always independent of the rotation angle of the ground motion. The maximum median drift along the x-direction is polarized in the FN direction (i.e., 90^o). Note that x-direction of the building coincides with 0^o. In contrast, the maximum median drift is polarized in the FP direction (i.e., 0^o). For other EDPs (chord rotation and internal forces), their maximum median values are generally independent of the rotation angle. This is also observed in Reyes and Kalkan (2012b) for single-story structures. It is clear that maximum median-EDPs are generally larger than the median MD-EDPs in the y-direction, while in the x-direction, maximum median-EDPs may be equal or smaller than MD-EDPs. These results clearly demonstrate that use of MD direction ground motions does not necessarily provide overconservative (or unrealistic) EDPs for systems responding in nonlinear-inelastic range in particular for asymmetric structures. Similar observations are valid for the symmetric building, as well.

From Figures 7 and 8, it is evident that conducting nonlinear RHA for ground motions oriented in the FN/FP directions does not always lead to the peak value of median-displacement over all non-redundant rotation angles. However, displacements are not underestimated substantially (less than 20%) if the buildings are subjected to only the FN/FP components of a large set of ground motions. Similar observations are valid for other EDPs investigated (Reyes and Kalkan 2012a).

7. CONCLUSIONS

The current state-of-practice in U.S. is to rotate the as-recorded pair of ground motions to the fault-normal and fault-parallel (FN/FP) directions before they are used as input for three-dimensional nonlinear response history analyses (RHAs) of structures. It is assumed that this approach will lead to two sets of responses that envelope the range of possible responses over all non-redundant rotation angles. Thus, it is considered to be a conservative approach appropriate for design verification of new structures. Based on the 9-story symmetric and asymmetric buildings, the influence that the angle of rotation of the ground motion has on several engineering demand parameters (EDPs) has been examined in nonlinear-inelastic domain. This investigation has led to the following conclusions:

1. Velocity-pulses may appear in directions different than the FN/FP directions.
2. The maximum drift over all non-redundant orientations seems to be polarized in the direction in which apparent velocity-pulse with period close to T_1 is observed; this polarization is almost perfect for symmetric-plan building.
3. There is no optimum orientation for a given structure maximizing all EDPs simultaneously; maximum value of EDP can happen in any direction different than the direction of the velocity pulse.
4. Conducting nonlinear RHA for ground motions oriented in the principal axes (that is, maximum direction) does NOT always lead to the maximum EDPs overall orientations for systems responding in nonlinear-inelastic range; this observation is true for both symmetric and asymmetric building examined here.
5. Conducting nonlinear RHA for ground motions oriented in the FN/FP axes does NOT always lead to the peak value of median-EDPs overall orientations. If only few ground motions are used, underestimations may be upto 20%.
6. It is shown that the proposed new ground motion definition (that is, MD) do not introduces overconservative bias to design ground motions as opposed to arguments in Stewart et al., (2011).

Although these observations and findings are primarily applicable to buildings and ground motions with characteristics similar to those utilized in this study, they are in close agreement with those reported in Reyes and Kalkan (2012a,b) and Kalkan and Kwong (2012a,b), where the influence of rotation angle on several EDPs has been examined using different structural systems.

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